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ABSTRACT

These instructional materials were produced as part of the project, Developing Computational Estimation Materials for the Middle Grades. The introduction to these sixth grade materials covers the following: why teach estimation; how the materials were developed; and how the lessons are organized. The 15 lessons that follow are designed to teach such estimation strategies as front-end estimation, compatible numbers, and rounding in lessons with whole numbers, fractions, and decimals. Each lesson plan includes objectives, teacher background, suggestions for teaching the lesson, acceptable answers for exercises, and six worksheets for student use. (MNS)

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Why Teach Computational Estimation?

Estimation has long been recognized as a valuable, useful skill in many vocations and in daily life. With the growing use of calculators and computers it is vital that people be able to judge the reasonableness of an answer. Also there are many instances where an estimate is all that is required to make an important decision. Despite the importance of estimation it has seldom received serious attention in curriculum materials and teachers have had few resources available for supplementing their own ideas. Evidence of students' performance on estimation indicates that most students do not have high proficiency with it, nor do even good estimators have a strong level of confidence in their ability to estimate.

How Were These Materials Developed?

In recent years there has been a renewed interest in this topic, including an increase in research on students' thinking on estimation tasks and on learning specific estimation strategies. This curriculum development project has been built upon this growing body of research. These materials were developed as part of a National Science Foundation project to provide a teaching resource for middle grades and junior high school. This particular set of lessons is designed to provide systematic instruction of effective estimation strategies in Grade 6. Other sets are available for Grades 7 and 8.

These lessons have been successfully used in schools. (A report documenting their effectiveness is available from any of the authors.) This package of lessons has been field tested and reflects helpful suggestions that have been provided by many teachers and students.

The extensive field testing of these materials revealed that there are many ingredients necessary to helping students become proficient estimators. One of these ingredients which you will want to keep in mind as you use these materials is the development of a proper mental set for estimation. This includes:

1. Recognition thatestimation is important and useful.
2. Awareness that many situations require only an estimate.
3. Recognition that there are many ways to obtain reasonable estimates.

How Are the Materials Organized?

Fifteen lessons have been written for each grade. Each lesson has the following components:

1. Objectives The objective(s) for each lesson is stated at the top of the first page of teacher notes.

2. Teacher Background This section discusses the strategies taught in the lesson in detail. Sometimes it also provides some additional background notes to help teachers better understand the approaches used.

3. Teaching the Lesson Brief suggestions for teaching the lesson are provided. The major portion of each lesson is developed through overhead transparencies. You will need to make the transparencies from the paper copies provided in these materials. The transparencies often provide real-world settings requiring estimation. They also present key steps highlighting each strategy along with examples for students to try under your direction. We think you will find the transparencies very useful in your teaching. They highlight the main ideas and focus students' attention on the key steps.

4. Using the Exercises Brief comments and suggestions for using the assignment sheets are given.

5. Student Assignment Sheets A two-page worksheet is provided for each lesson. Each worksheet also provides some real-world applications of estimation. These should be started in class and completed as homework. As time permits, discussion of selected exercises the following day will promote estimation thinking and awareness of the many ways of obtaining a reasonable estimate.

What Are The Lessons?

The lesson titles for the sixth grade materials are given here. In most cases the titles are descriptive, however please refer to the specific lessons for a more comprehensive explanation of the topic.

GRADE 6

TABLE OF CONTENTS

Lesson 1: *Front-End Addition Estimation*

Lesson 2: *Adjusting Front-End Estimates*
Compatible Numbers: Sums Close to "Nice" Dollar Amounts

Lesson 3: *More Adjusting Front-End Estimates*
(Horizontal Form)

Lesson 4: *Front-End Subtraction Estimation*

Lesson 5: *Estimate Sums and Differences of Larger Numbers*

Lesson 6: *Multiplication Estimation with Adjusting: One Factor Less than Ten*

Lesson 7: *Multiplication Estimation with Adjusting: Rounding, Use of 10, 100, and 1000*

Lesson 8: *Division Estimation - One-Digit Divisors: Size of Quotient, Estimate Quotients*

Lesson 9: *Division Estimation - One-Digit Divisors: Compatible Numbers*

Lesson 10: *Division Estimation - Two-Digit Divisors: Compatible Numbers*

Lesson 11: *Division Estimation - Two-Digit Divisors: Rounding of Divisor*

Lesson 12: *Estimate Fractional Amounts*
Identifying Fractions Close to 0, ½, and 1

Lesson 13: *Estimate Sums of Fractions and Mixed Numbers*

Lesson 14: *Estimate Fractional Parts: Compatible Numbers*

Lesson 15: *Estimate Sums and Differences of Decimals*

USING THE MATERIALS

The process of developing students' estimation competency is a long one. As they have repeated contacts with estimating and as they develop competence with specific techniques for obtaining an estimate, students will gain skill and confidence. Although your students may not reach a high level of competency in one year, progress will be made through systematic instruction.

You have an important role to play in developing students' ability to estimate. Initially many students may show resistence toward estimating. Other students will welcome the opportunity to share self-developed estimation strategies. Through discussion of thinking strategies with students and the encouragement of students' sharing their own thinking for a problem, you can help them gain new appreciation for the estimation process.

We think these lessons emphasize the important components of estimation skill and will be most interested in learning about your experience in using them. Good luck to you!

NSF ESTIMATION
GRADE 6 - LESSON 1

OBJECTIVES: To create an awareness and appreciation for the usefulness of estimation.

To introduce the front-end strategy with informal adjusting of the initial estimate.

TEACHER BACKGROUND:

The lesson introduces the nature and usefulness of estimation and presents the front-end addition strategy as one effective way to estimate. In the lesson and throughout the program, students should recognize and accept that:

- estimation is important;
- many times only an estimate is required;
- there are many ways of obtaining an appropriate estimate;
- any estimate within a reasonable range is acceptable;
- estimation should be done quickly and mentally, and produce reasonable answers.

Students need to gain confidence in their ability to estimate and become flexible in their estimation thinking. To attain these goals, it is important to engage them in discussion and listen to how they think.

This is the first of three lessons on the FRONT-END estimation strategy with addition. This is a useful and efficient estimation technique. In many situations it is preferable to rounding, since the addends are visible. The technique consists of first finding the lower bound and then adjusting the initial estimate upward.

FRONT-END estimation begins by finding the sum of the lead (or front) digits. At the right this sum is \$8, which is an initial estimate.

$4.68
1.19
+ 3.45

FRONT-END SUM: \$4 + \$1 + \$3 = \$8

INITIAL ESTIMATE: \$8 OR \$8+

An examination of the cents shows that they total more than a dollar so the initial estimate is ADJUSTED upward to \$9, over \$9 or \$9+.

$4.68
1.19
+ 3.45

ADJUSTED ESTIMATE: \$9 or over \$9

6-1-1

TEACHING THE LESSON

GET YOUR MIND IN GEAR

Each lesson begins with a transparency designed to help students think about the nature and process of estimation. TR #1 presents three situations where estimation is being used. As you discuss each one with students, ask questions such as;

Shopper Does the shopper have enough money?
How do you know?
Which two things could he buy?
Do you ever estimate in situations like this?

Engineer How do you think she arrived at her estimate?
(By estimating how long each part of the job would take.)

Why is estimating important here?
(To figure the cost of the whole job.)

Teacher What would be a good estimate? (90)
How do you know? (All the numbers are close to 90)
How close do you think 90 is to the actual average?

Indicate that several words are used when estimating, such as about. Have students suggest other words that are associated with estimating. Suggest one or two to get them started.

nearly	almost	about
around	close to	a little more (less) than
approximately	over	under

FRONT-END ADDITION

Point out that students are going to learn one way to estimate and will be working with other ways as the estimation work continues.

TR #2: Use this to introduce the front-end approach. Here students find only the sum of the dollars. This tells that the actual amount is over this sum. If students want to be more precise, accept their ideas.

Emphasize the three features in the middle of the transparency:

DONE QUICKLY - as time is usually limited

DONE MENTALLY - as paper and pencils are often not accessible

PRODUCES A REASONABLE ANSWER - as this is all that is needed many times.

Tell students to keep these features in mind to help them better understand estimation and its usefulness.

Use the TRY THESE exercises. If students try to get closer with their estimates, that is acceptable.

ANSWERS: $8.00 $12.00 $9.00 $16.00

TR #3: Adjusting upward is introduced. Note the idea of "getting closer." Here students just need to find if the number of cents make it go over another dollar.

ANSWERS 1. $9.00+ YES 2. $7.00+ YES NO
3. $8.00+ YES YES

TR #4: Now front-end estimation is practiced in a situation involving a known amount or reference point. The situation is a real world one that supports the use of estimation. After discussing each situation at the top, have several students tell how they found their answers for the four exercises at the bottom.

ANSWERS 1. FOR SURE 2. NO WAY
3. HARD TO TELL 4. FOR SURE

USING THE EXERCISES

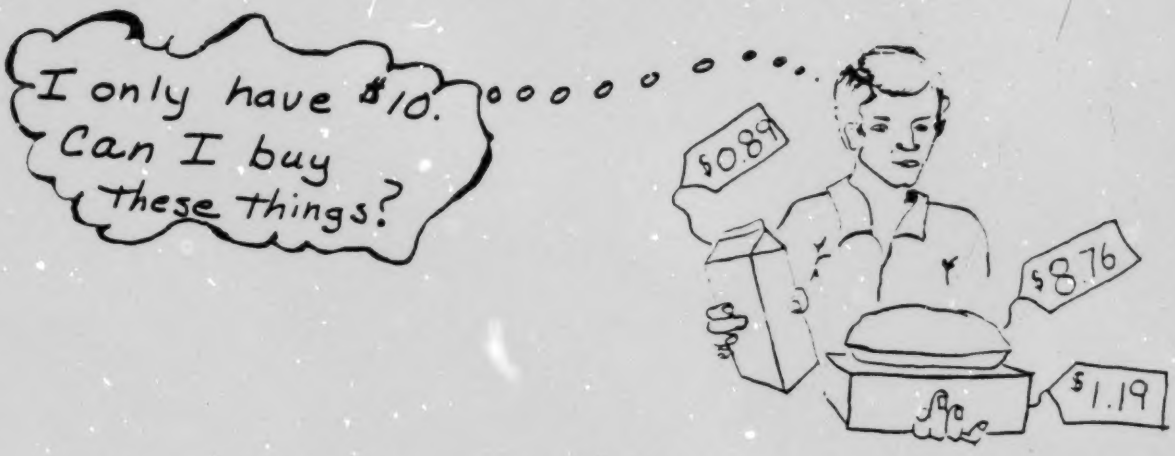
With the lesson development as a background, students should be ready for independent work. You might do one or two examples of each type with them so they will know what is expected. Emphasize that they do not need to find exact answers and encourage them to estimate.

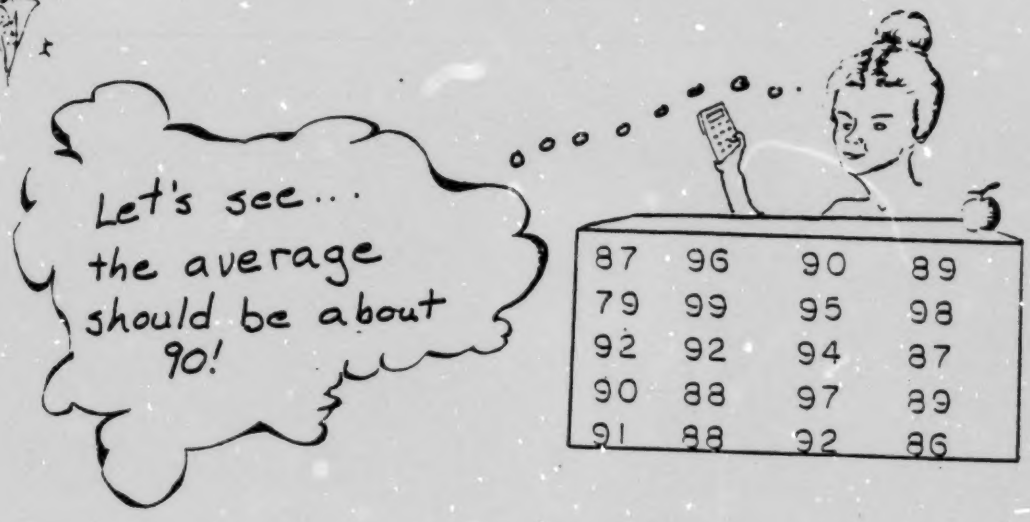
ANSWERS

1. $7.00, under $8.00
2. $9.00, over $10.00
3. $7.00, under $8.00
4. $14.00, over $15.00
5. $7.00, over $8.00
6. $12.00, over $14.00
7. over $6.00
8. under $9.00
9. over $10.00
10. over $8.00
11. $11.00
12. $15.00 or $16.00
13. $12.00 or $13.00
14. $8.00 to $10.00
15. NO
16. YES
17. YES
18. NO
19. YES
20. YES
21. NO
22. NO

GET YOUR MIND IN IN GEAR

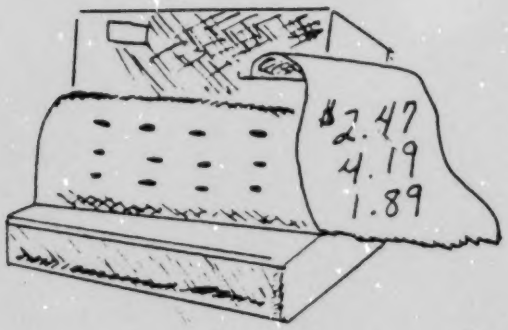
USING ESTIMATION





6-1-TR1

FRONT-END ESTIMATION



Add the front-end:

$2.47
4.19
1.89

Estimate:

OVER $7

OR

$7+

It's Quick.

It's faster than paper and pencil! It's faster than punching the numbers in a calculator!

It's Reasonable:

$7 is in the right "BALLPARK"! What is the most you can be off?

It's Done in your Head:

You can easily add 2+4+1 in your head.

TRY THESE:

$4.68	$6.03	$5.49	$8.19
1.19	2.51	1.16	2.40
+ 3.45	+ 4.27	+ 3.42	+ 6.82
OVER $___.00	OVER $___.00	OVER $___.00	OVER $___.00

6-1-TR2

GETTING CLOSER

$2.41
6.15
1.89

$_.00

Front-End Estimate:

$ _.00^{+}

Look at the rest:

$ 2.41
6.15
1.89

If the total of the cents goes over #1, the sum will be over #10.

WILL THE CENTS MAKE IT GO OVER $10.00? _____

$5.19
.21
.08
2.12

$_.00

Front-End Estimate:

$ _.00^{+}

Getting Closer:

UNDER $8.00? _____

OVER $8.00? _____

$5.89
2.91
.37
1.62

$_.00

Front-End Estimate:

$ _.00^{+}

Getting Closer:

OVER $9.00? _____

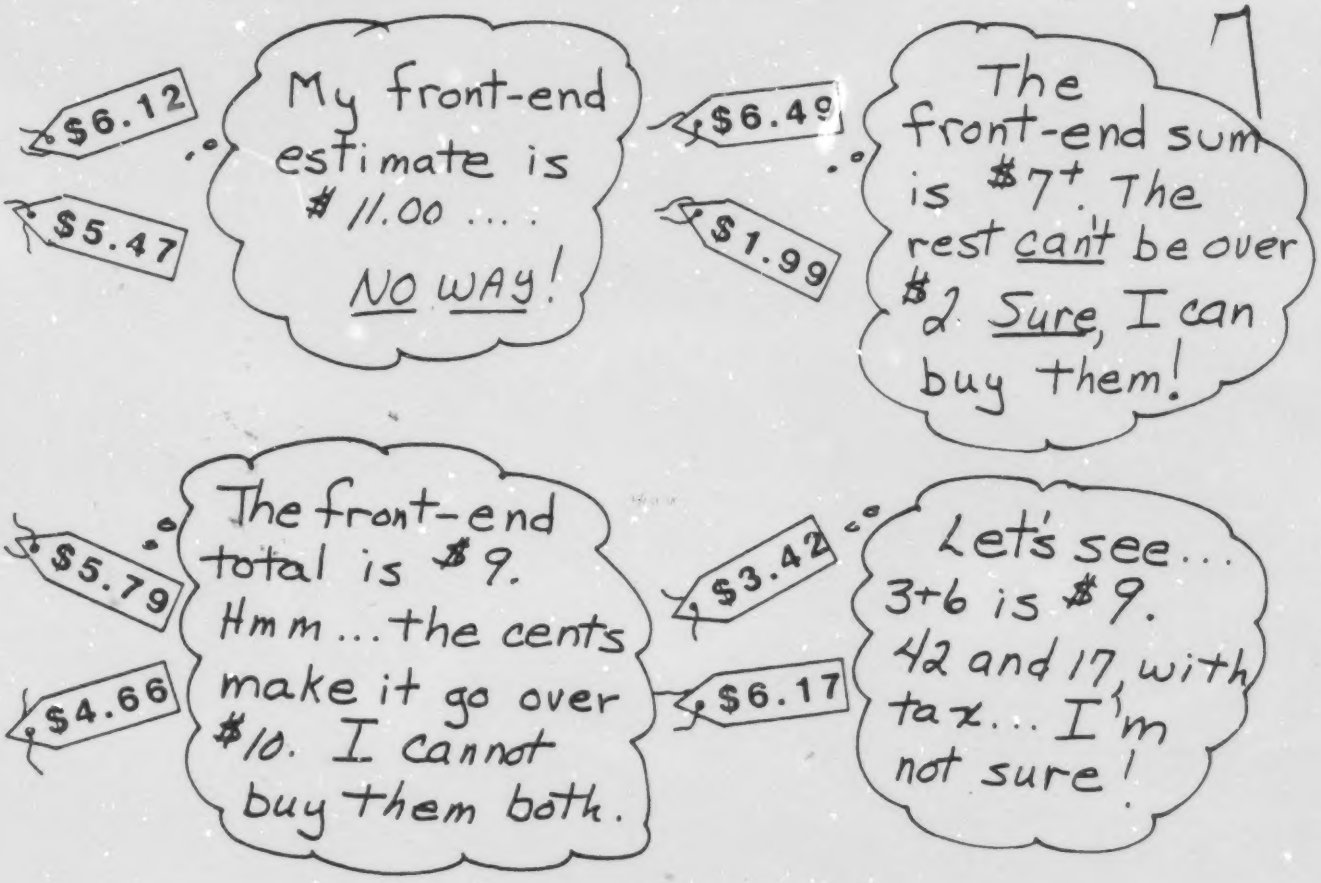
OVER $10.00? _____

How can you tell?

6-1-TR3

IS $10.00 ENOUGH?

Do you have enough to buy...


$6.12
$5.47
My front-end estimate is $11.00 .... NO WAY!
$6.49
$1.99
The front-end sum is $7+. The rest can't be over $2. Sure, I can buy them!
$5.79
$4.66
The front-end total is $9. Hmm... the cents make it go over $10. I cannot buy them both.
$3.42
$6.17
Let's see... 3+6 is $9. 42 and 17, with tax... I'm not sure!

TRY THESE: **Is $10.00 enough money?**
Think about tax...

CIRCLE ONE.

$6.12	$2.89	FOR SURE	HARD TO TELL	NO WAY
$4.15	$6.28	FOR SURE	HARD TO TELL	NO WAY
$3.47	$6.28	FOR SURE	HARD TO TELL	NO WAY
$2.27	$7.02	FOR SURE	HARD TO TELL	NO WAY

6-1-TR4

Name _____

Find the front-end sum.

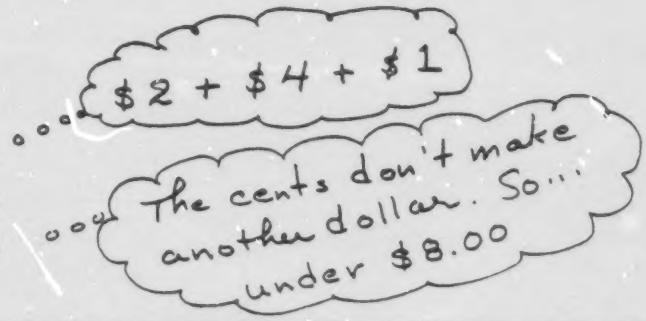
Circle the better choice for getting closer.

1.

$2.20
4.15
+ 1.35
//////

Front-End Estimate: $_____

Getting under $8.00?
Closer: over $8.00?



2.

$6.48
2.97
+ 1.17
//////

Front-End Estimate: $_____

Getting under $10.00?
Closer: over $10.00?

3.

$4.29
+ 3.45
//////

Front-End Estimate: $_____

Getting under $8.00?
Closer: over $8.00?

4.

$7.19
2.65
+ 5.79
//////

Front-End Estimate: $_____

Getting under $15.00?
Closer: over $15.00?

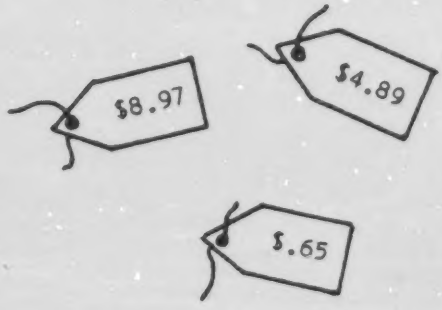
5.

$1.15
3.29
1.62
+ 2.24
//////

Front-End Estimate: $_____

Getting under $8.00?
Closer: over $8.00?

6.



Front-End Estimate: $_____

Getting under $14.00?
Closer: over $14.00?

6-1-p.1

Name_____

Circle the best estimate.

7.
$3.35
.75
+ 2.10
———
//////

under $6.00 ?

over $6.00 ?

8.
$7.10
1.25
.15
+ .20
———
//////

under $9.00 ?

over $9.00 ?

9.
$4.98
1.93
.21
+ 3.15
———
///////

under $9.00 ?

over $10.00 ?

10.
$5.37
.15
2.97
+ .20
———
/////

over $8.00 ?

over $9.00 ?

Estimate. Use the front-end method.

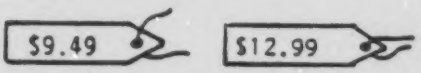
11.
$4.49
2.30
+ 5.06
———
over_____

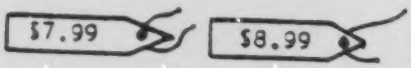
12.
$8.38
3.49
.62
+ 4.15
———
over_____

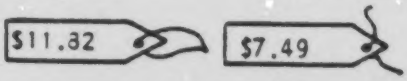
13.
$6.27
5.85
+ 1.18
———
over_____

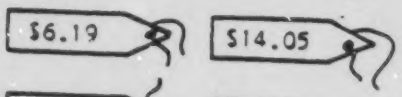
14.
$4.95
1.67
+ 3.98
———
over_____

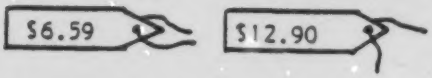
Is $20.00 enough? Circle the price tags of the pair of things that you have money to buy. Assume the sales tax is included in the price.

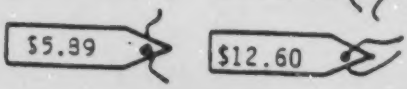
15. 

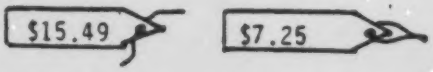
16. 

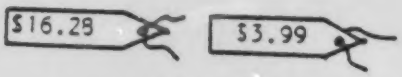
17. 

18. 

19. 

20. 

21. 

22. 

6-1-p.2

NSF ESTIMATION
GRADE 6 - LESSON 2

OBJECTIVES: To estimate by grouping numbers whose sum is close to $1.00.

To use grouping to adjust upward for front-end estimation.

To estimate by grouping numbers to dollar amounts that are easy to work with mentally.

TEACHER BACKGROUND:

Estimation is a flexible process in which the approach used often depends on the numbers and the situation. It cannot be reduced to a single step-by-step process. You can expect that many students will initially feel uncomfortable with estimation. However, as they gain more experience with various approaches, and have the opportunity to discuss the work, they will become more comfortable.

The lesson has three parts:

1. First, students look for amounts that are close to $1.00 and use this to estimate.

   Note: Final estimates will vary. Any estimate between $1.20 and $1.30 is good.

a. $.38 and $.64 are about $1.00

b. $.23 more makes it about $1.20 to $1.25

2. Grouping numbers to $1.00 is used to adjust the initial estimate upward.

   While grouping produces more precise estimates, it is still acceptable to give the estimate as "over $10.00" or "$10.00+"

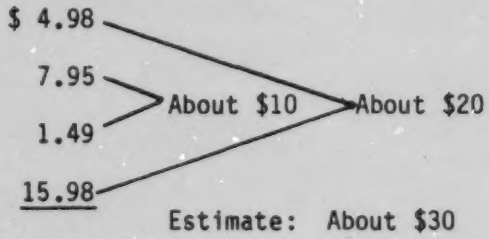
$6.43
.88
+ 3.13

a. FRONT-END SUM: $9.00

b. $. 88 and $.13 is about $1.00

c. FINAL ESTIMATE: $10.40

3. Grouping numbers to "nice" dollar amounts is an extension of grouping cents to dollars. Here, one looks at the numbers and groups them to whole dollar amounts that are easy to work with.



As you teach the lesson, be sure to allow time for discussion and encourage students to suggest other approaches.

TEACHING THE LESSON

GET YOUR MIND IN GEAR

TR #1 addresses the theme of a rough initial estimate (close enough) versus the need to refine an initial front-end estimate (getting closer). Students need to develop an awareness about when "eyeballing" is enough and when they need to get closer.

Have students begin each time by finding the sum of the dollars (lead digits). Encourage them to share their thinking.

ANSWERS: Top Left: Not enough - tell at a glance (eyeball)

Top Right: Hard to tell - need to get closer (refine)

Bottom Left: Not enough - need to get closer (refine)

Bottom Right: Enough - tell at a glance (eyeball)

FRONT-END AND GROUPING

TR #2: Discuss the top of TR #2. Emphasize that they are to look for amounts that "go together" to make about $1.00. Use the two TRY THESE exercises to give additional practice.

The work is extended at the bottom to grouping two numbers to $1.00 and then adding the third number to $1.00 to get an estimate. Estimates will vary as a result of the approach used.

ANSWERS: TOP: $2.00, $3.00

BOTTOM: $1.50 - $1.60; $1.10 - $1.25; $1.80 - $1.90

TR #3: Here grouping cents to a dollar is used to adjust front-end estimates. Note the distinction between close enough and getting closer. Stress that grouping helps one to get closer. Use the getting closer approach in the TRY THESE exercises. Allow a variety of estimates that are within a reasonable range.

ANSWERS: $9.80 - $10.00 $4.20 - $4.40

TR #4: Have students suggest pairs of numbers that group to the amounts on the left (TOP). Then show how this can be used to estimate and have students apply this with the TRY THESE exercises.

ANSWERS: $24.00+ - $26.00

$39.00 - $42.00

$16.00 - $19.00

As you conclude the lesson remind students that there are many ways to estimate. They need to select their own approach and decide on the level of precision needed. "Close enough" estimates are often fine. Also, encourage them to study a situation for a few minutes before starting to estimate. This permits them to analyze the situation and find an appropriate approach that is quick and can be done easily in their heads.

USING THE EXERCISES

Discuss the two sample exercises on page 1 as well as exercise 10 with students before they start the written work. Also discuss what they are to do on exercise 16 - 21 and 22 - 25. Emphasize that they are to estimate. Exact sums are not needed.

ANSWERS:

1. $.79 and $.18 $.52 and $.47	2. $.39 and $.59 $.22 and $.81	3. $.74 and $.23 $.12 and $.89
4. $1.00 - $1.10 ($1.00+)	5. $1.90 - $2.20 ($2.00+)	6. $1.20 - $1.30 ($1.00+)
7. $1.40 - $1.60 ($1.00+)	8. $1.90 - $2.10	9. $2.25 - $2.50 ($2.00+)
10. $8.00 - $8.30 ($8.00+)	11. $8.00 - $8.20 ($8.00+)	12. $11.00 - $11.50 ($11.00+)
13. $10.00 - $10.25 ($10.00+)	14. $9.00 - $10.00 ($9.00+)	15. $11.00 - $11.50 ($11.00+)
16. $4.15 and $5.97	18. $10.99 and $4.15	20. $14.10 and $5.97
17. $8.95 and $1.09	19. $8.95 and $5.97 $14.10 and $1.09	21. $10.99 and $8.95
22. $19 - $21	23. $34 - $36	24. $42 - $45

25. Answers will vary.

GET YOUR MIND IN GEAR

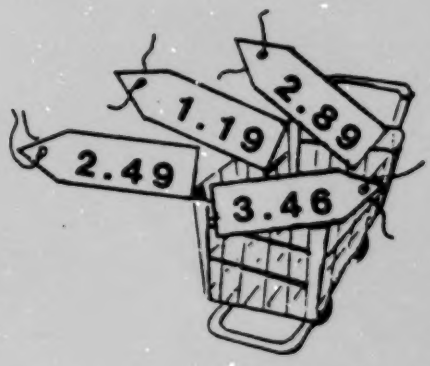
Sometimes you only need to "eyeball" an estimate ... sometimes you need a more refined estimate.

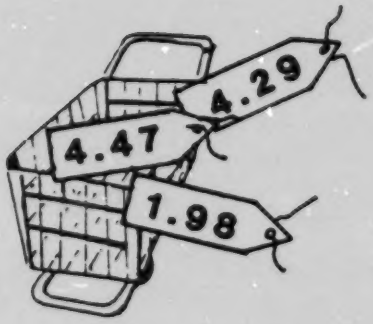
DECIDE:

EYEBALL: Can you tell if you have enough in a glance?

OR

REFINE: Do you need to get a little bit closer?

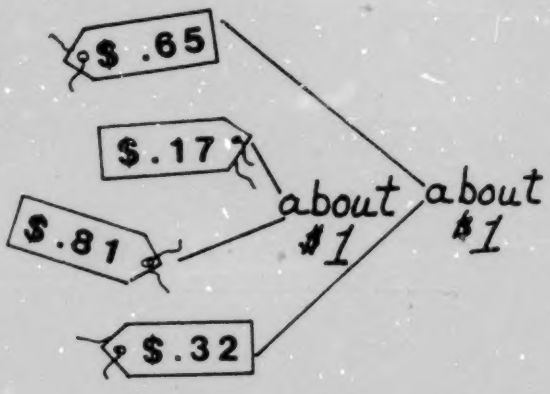




6-2-TR1

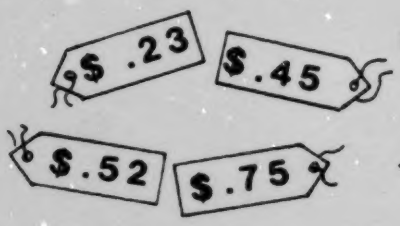
GROUPING CENTS TO DOLLARS

You can estimate by grouping cents to dollars.

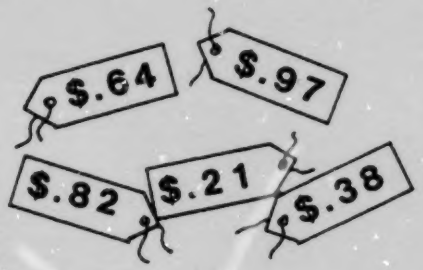


Estimate:

TRY THESE:



Estimate: _____

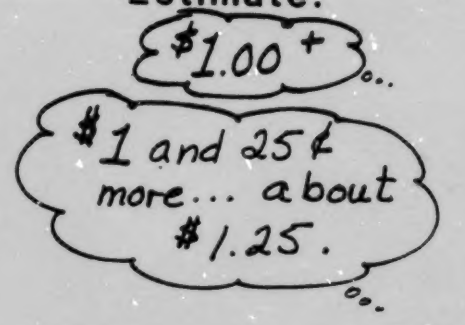


Estimate: _____

Now try to refine your estimate.

$ .38
.23
+ .64
about #1

Estimate:



TRY THESE:

$ .27	$ .34	$ .83
.73	.19	.44
+ .52	+ .68	+ .61

Estimate: _____ _____ _____

6-2-TR2

ADDING DOLLARS & GROUPING CENTS

To estimate the total, you can combine the front-end method with grouping cents.

$6.43
.88
3.13

Add the dollars:

$6.43
.88
3.13
—.—

Group the cents:

$6.43
.88 → about $1
.13

1+
$1.40
____

Put them together:

CLOSE ENOUGH

Front-end... $9
Cents ... $1+
I'll say over $10

GETTING CLOSER

Front-end ... $9
Cents ... $1.40
I'll say $10.40

TRY THESE:

$ 7.39
.67
1.88

FRONT-END:
CENTS ...:
ESTIMATE :

$1.19
2.82
.31

FRONT-END:
CENTS ...:
ESTIMATE :

6-2-TR3

GROUPING TO NICE NUMBERS

Name 2 numbers that group to:

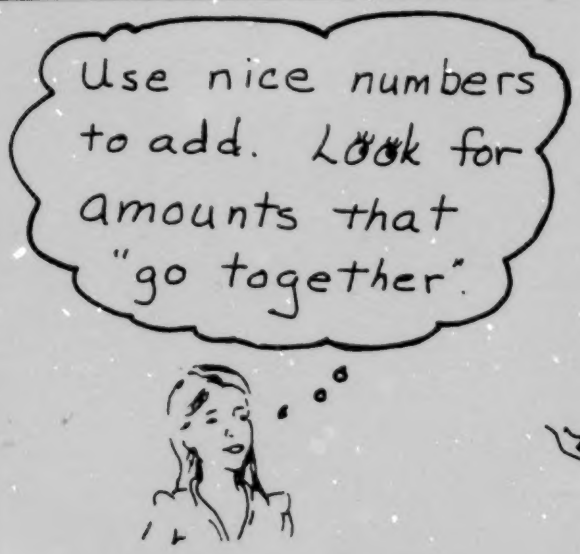
$4.15 $6.05

About $10.00 $4.09

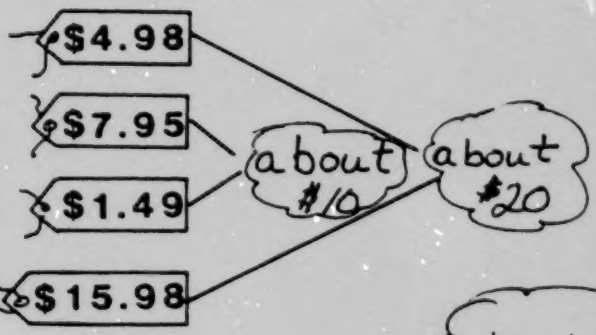
$17.95

About $20.00

About $30.00 $5.99 $13.16



ESTIMATE THE TOTAL:



Estimate: about $30

TRY THESE:

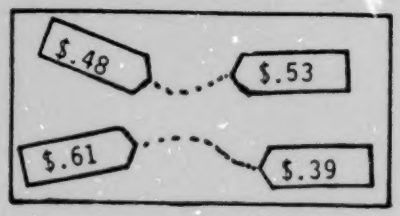
6-2-TR4

Name _____

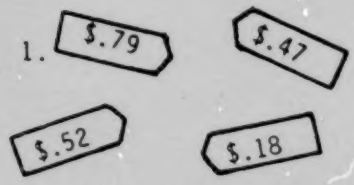
GROUP TO DOLLARS

Draw lines between pairs of price tags that make about $1.00.

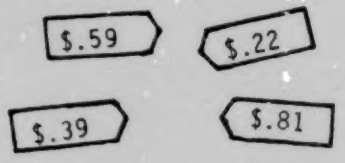
Ex.



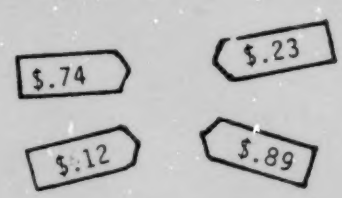
1.



2.



3.



Estimate the total. First look for amounts that make a dollar. Then get closer with your estimate.

Ex.

$.62
.55
+ .39
/////

about $1.00
50¢ more

Estimate: $1.50

4.

$.23
.21
+ .63
/////

Estimate: _____

5.

$.98
.75
+ .28
/////

_____

6.

$.49
.27
+ .49
/////

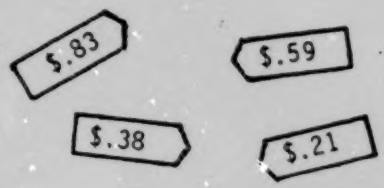
_____

7.

$.33
.69
+ .49
/////

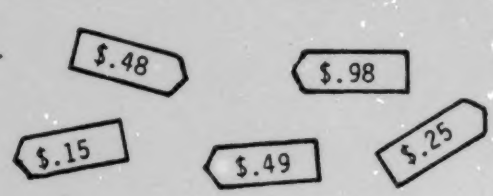
_____

8.



Estimate: _____

9.



_____

FRONT-END AND GROUPING

Estimate. 1. Find the front-end sum.
2. Group cents to dollars and adjust to get closer
3. Write the estimate.

10.

$5.45
.25
+ 2.53
/////

Front-end: _____

Estimate of cents: _____

Final Estimate: _____

11.

$3.15
4.38
+ .63
/////

Front-end: _____

Estimate of cents: _____

Final Estimate: _____

6-2-p.1

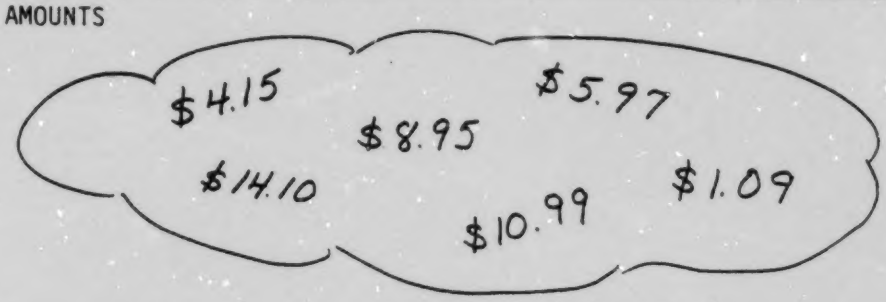
Name _____

Estimate by first finding the front-end sum.

12.	13.	14.	15.
$8.18	$ .77	$4.89	$7.28
.79	3.12	4.60	.49
+ 2.45	+ 6.24	+ .12	+ 3.72
//////	//////	//////	//////
Estimate: _____	_____	_____	_____

GROUP TO NICE DOLLAR AMOUNTS

Find pairs of prices that group to:



About $10.00	About $15.00	About $20.00
16. _____ and _____	18. _____ and _____	20. _____ and _____
17. _____ and _____	19. _____ and _____	21. _____ and _____

SPORTS SALE	
Sport shoes	$23.95
Sweat socks	3.89
Basketballs	15.39
Baseballs	2.49
Baseball bats	6.38
Tennis Racquets	17.29

Estimate: Group to nice numbers if you can.

22. 1 Basketball
    2 Baseballs About: _____

23. 1 Sport shoes
    1 Baseball bat
    1 Sweat socks About: _____

24. 1 Tennis Racquet
    1 Sport shoes
    1 baseball About: _____

25. Find 3 things that cost close to $15.00. You can buy more than one of each item.

_____ _____ _____

6-2-p.2

NSF ESTIMATION
GRADE 6 - LESSON 3

OBJECTIVES: To recognize reasonable amounts.

To adjust front-end estimates for 3- and 4-digit numbers.

To use front-end estimation for horizontal examples.

TEACHER BACKGROUND:

Lesson 3 focuses on adjusting the front-end sum in order to get a more precise estimate with 3-digit and 4-digit numbers. A variety of ways can be used to adjust the initial estimate. Two approaches are shown below.

FRONT-END SUM: Find the sum of the lead digits and indicate the place value.

289
416
+ 162

(2 + 4 + 1) hundreds = 7 hundreds

INITIAL ESTIMATE: 700

ADJUSTING:

1. Find the sum of the tens digits and adjust up.

(8 + 1 + 6) tens is 15 tens or 150.

700 + 150 = 850

ESTIMATE: 850

2. Find numbers that group to 100.

89 + 16 is about 100. 62 more.

700 + 160 = 860

ESTIMATE: 860

The lesson does not emphasize one particular method for the adjusting step. Other methods may be suggested by students also. It is important that you do not press for precise estimates at this time for all students. (Over 800, 800 or 800+ are still acceptable estimates.) Some students may not possess all the prerequisite skills for adjusting estimates as shown above. A long range goal is to have students view estimation as something that is practical, useful and relatively easy to do mentally.

NOTES: 1. Estimates are acceptable as long as they are within a reasonable range. For example, in the above example, any estimate from 800 to 900 is acceptable.

2. The background established by the first two lessons, which involved money and adjusting informaliy, should have established a solid base for Lesson 3.

TEACHING THE LESSON

GET YOUR MIND IN GEAR

The focus of TR #1 is sensible answers. One long term value of work on estimation should be greater sensitivity to answers that are not sensible or reasonable. Discuss each exercise with the students. In the exercises at the bottom, you may want to have students look up information to determine a reasonable answer.

ANSWERS TOP: $14.00 500 $8.00

BOTTOM: Answers will vary.

ADJUSTING FRONT-END ESTIMATES

TR #2: Present the three steps shown on the transparency. Discuss ways of doing step 2, including those discussed in TEACHER BACKGROUND. In the TRY THESE exercises have students find two estimates: close enough and getting closer.

ANSWERS: 1300^{+} to 1400 1100^{+} to 1160 1200^{+} to 1300

1400^{+} to 1500 2000^{+} to 3000

TR #3: Here the work is extended to 4-digit numbers. For adjusting the estimate discuss how the sum of the hundreds digits can be used as well as grouping hundreds.

ANSWERS: $11{,}000^{+}$ to 12,000 $12{,}000^{+}$ to 13,000

$9{,}000^{+}$ to 10,000 $11{,}000^{+}$ to 12,000

HORIZONTAL EXAMPLES

TR #4: Focus on the importance of the lead digits having the same place value. In the next lesson the examples have different numbers of digits.

ANSWERS: $7{,}000^{+}$ to 7,500 $2{,}000^{+}$ to 2,400 $16{,}000^{+}$ to 16,500

900^{+} to 950 $5{,}000^{+}$ to 5,500

USING THE EXERCISES:

Develop exercises 1 and 4 on page 1 with the students. On page 2 students get practice in finding and selecting sensible amounts. Remind students to estimate on exercises 20 -23.

ANSWERS:

1. 1200+ to 1300	2. 1400+ to 1500	3. 10,000+ to 11,000
4. 1200+ to 1300	5. 9000+ to 10,000	6. 1200+ to 1300
7. 12,000+ to 13,000	8. 1100+ to 1200	9. 12,000+ to 14,000
10. 1200+ to 1400		

11. will vary	12. will vary	13. 10 - 12 years old
14. 300 - 450	15. will vary	16. $25.00
17. 25	18. $15.00	19. 300
20. $7.50	21. $21.00	22. 600
23. $9.75		

YOUR MIND IN GEAR

WHAT'S SENSIBLE?

Choose the reasonable number:

A NEW BASEBALL GLOVE COSTS:

$1.40 $14.00 $140.00

THE NUMBER OF STUDENTS WHO ATTEND WASHINGTON SCHOOL IS:

5 50 500

THE GONZALES FAMILY OF 4 WENT TO SPEEDY BURGER FOR LUNCH. THEY SPENT ABOUT:

$8.00 $48.00 $80.00

Fill in a reasonable amount:

IT'S ABOUT _____ MILES FROM HERE TO CALIFORNIA.

A SMALL COMPACT CAR GETS ABOUT _____ A GALLON.

THERE ARE ABOUT _____ STUDENTS IN OUR SCHOOL.

THE AVERAGE NUMBER OF PEOPLE IN A FAMILY IS _____.

THERE ARE ABOUT _____ PEOPLE LIVING IN THE UNITED STATES.

6-3-TR1

ADJUSTING FRONT-END ESTIMATION

Estimate The Number Of Students

Attending The Three Schools.

SCHOOL	STUDENTS
WASHINGTON	378
KING	236
JEFFERSON	442

REMEMBER: WITH FRONT-END YOU BEGIN WITH THE DIGITS REPRESENTING THE LARGEST PLACE VALUE!

Add the hundreds: _____

Estimate the rest: OVER 100? OVER 200?

Put it together:

CLOSE ENOUGH

9 hundred + 100+ ... over 1000 !

GETTING CLOSER

378 } about 150.
236 }
+442 } I'll say 1050.

TRY THESE

329	627	287	863	772
562	119	163	+619	852
+489	+406	401		+467
		+428		

6-3-TR2

MORE ADJUSTING

GAME ATTENDANCE

GAME 1	3,529
GAME 2	2,907
GAME 3	3,273

Add the thousands: _____

Estimate the rest: OVER 1000? OVER 2000?

Put it together: _____

CLOSE ENOUGH

8 thousand + 1000+
... over _____

GETTING CLOSER

3,529
4,907
+ 3,273
} about 1600

about _____

TRY THESE:

1,2 2 5	3,6 8 3	6,7 6 2	$ 3,2 6 3
3,1 7 9	5,0 0 6	+ 2,5 3 7	6,8 9 1
2,6 1 9	+ 4,2 0 2		+ 1,7 2 5
+ 4,3 5 8			

6-3-TR3

HORIZONTAL FORM

When examples are written horizontally:

Make sure the front-end digits have the same place value:

Each digit shows thousands.

8,427 + 4,672 + 3,820

Then, estimate as usual

FRONT-END SUM: _____

FINAL ESTIMATE: _____

TRY THESE

2,356 + 1,582 + 3,424

FRONT-END SUM: _____

FINAL ESTIMATE: _____

427 + 304 + 683 + 976

9,007 + 4,468 + 2,796

103 + 482 + 326

3,265 + 2,123

6-3-TR4

NAME_____

Estimate. Use front-end addition and adjust.

1.
$$\begin{array}{r} 6{,}247 \\ 2{,}728 \\ +\ 3{,}563 \\ \hline /////\end{array}$$

a. Sum of thousands: _____

b. Estimate of rest:

under 1000 over 1000 over 2000

c. Final Estimate: $12{,}000^{+}$ or about _____

2.
$$\begin{array}{r} 419 \\ 207 \\ 526 \\ +\ 332 \\ \hline ///\end{array}$$

a. Sum of hundreds: _____

b. Estimate of rest:

under 100 over 100 over 200

c. Final Estimate: 1400^{+} or about _____

3.

$2{,}875 + 1{,}625 + 5{,}902$

a. Sum of thousands: _____

b. Estimate of rest:

under 1000 over 1000 over 2000

c. Final Estimate: _____$^{+}$ or about _____

Estimate. You can get close enough or try to get closer.

Ex.
$$\begin{array}{r} 347 \\ 236 \\ +\ 670 \\ \hline ///\end{array}$$
Carl: 1200^{+}

Anne: 1240

4.
$$\begin{array}{r} 847 \\ 129 \\ +\ 315 \\ \hline ///\end{array}$$
Est: _____

5.
$$\begin{array}{r} 6{,}275 \\ +\ 2{,}939 \\ \hline /////\end{array}$$
Est: _____

6.
$$\begin{array}{r} 357 \\ 106 \\ 572 \\ +\ 245 \\ \hline ///\end{array}$$
Est: _____

7. $2{,}461 + 7{,}129 + 3{,}291$

Estimate: _____

8. $862 + 319$

Estimate: _____

9. $1{,}346 + 2{,}567 + 9{,}356$

Estimate: _____

10. $363 + 478 + 268 + 201$

Estimate: _____

6-3-p. 1

Name_____

Estimation involves recognizing and finding sensible amounts.
Fill in an amount that makes sense.

11. The population of the city you live in: _____

12. The time it takes to clean your room: _____

13. The age of a sixth grade student: _____

14. The number of miles traveled in a 7-hour car trip: _____

15. The number of multiplication facts you can do in 1 minute: _____

Circle the most sensible choice for each exercise.

16.	A pair of school shoes costs:	$2.50	$25.00	$250.00
17.	The number of students in Mrs. Smith's class is:	3	12	25
18.	The Wong family (mother, father, two children) went to the movies one night. They spent about:	$1.50	$15.00	$40.00
19.	A jumbo jet was almost filled with passengers. The number of people on the plane was about:	300	600	900

Use addition estimation to choose the closest estimate.

20.	Larry is sending for a hobby kit that costs $6.50 plus 95¢ for postage. He should send about:	$6.50	$7.50	$8.50
21.	Ms. Paine bought a blouse for herself that cost $12.95 and one for her daughter that cost $7.49. She spent about:	$19.00	$21.00	$23.00
22.	Washington School has 202 fourth graders, 194 fifth graders and 189 sixth graders. The total number of students is about:	600	650	700
23.	Ms. Conley has to pay bills of $2.55, $3.42 and $3.78. She has to pay about:	$7.75	$8.75	$9.75

NSF ESTIMATION
GRADE 6 - LESSON 4

OBJECTIVES: To use front-end estimation for subtraction.

To estimate when exercises are written horizontally and the numbers have different numbers of digits.

TEACHER BACKGROUND:

Front-end estimation is now extended to subtraction as shown below:

$$\begin{array}{r} 9327 \\ -\ 6875 \\ \hline \end{array}$$

1. Subtract lead digits to get an initial estimate.

$$\begin{array}{r} 9 \text{ thousand} \\ -\ 6 \text{ thousand} \\ \hline 3 \text{ thousand} \end{array}$$

2. Adjust the initial estimate

$$\begin{array}{r} 9327 \\ -\ 6875 \\ \hline \end{array}$$

327 is less than 875 so the answer is less than 3000.

ESTIMATE: 3000

To adjust the initial estimate (3000), it is possible to just compare the hundreds digits.

Getting closer: The estimate can be made more precise by actually subtracting the hundreds digits. 13 hundred - 8 hundred = 5 hundred, so the estimate is 2500.

However, at this level it is important to avoid pressing for estimates that are too precise. More capable estimators should be allowed to give more precise estimates and there is a section of exercises on page 2 of the worksheets that deals with this.

The lesson also presents examples written horizontally in which the numbers have different numbers of digits. Students need to remember to use digits of equal place value when estimating.

427 + 89 + 279 Front-End Sum: 600

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 stresses flexibility in estimation. There are several ways to approach the two grocery lists shown on the transparency.

$3.19	For the example at the left, one could begin by using front-end estimation and then adjusting up. The adjusting could be done by adding the dimes or grouping cents to dollars.
.84	
.45	
1.39	
.08	
2.98	Another approach is to group the first two amounts to $4.00, the next three amounts to $2.00 and rounding $2.98 to $3.00.

Let students share their approaches to the two exercises.

ANSWERS: $8.00 to $9.50 $3.00 to $4.00

FRONT-END SUBTRACTION

TR #2: Discuss the two examples, emphasizing the adjusting step. If some students want to make more precise estimates on the TRY THESE exercises, have them share their thinking with the class.

ANSWERS: under 600 over 200 under $5.00

TR #3: The work is now extended to 4-digit numbers. Have students show the adjusting by writing + or - after the front-end difference.

ANSWERS: 5000^{+} 2000^{-} 1000^{+}

HORIZONTAL EXERCISES

TR #4: Discuss the work on the top half of the transparency, emphasizing that they must work with digits which have the same place value.

ANSWERS: 8000^{+} to 9000 2000^{+} to 2100

550 to 700 4000^{-}

USING THE EXERCISES:

Discuss the sample exercises that occur before exercise 7 on page 1 and exercises 17 and 21 on page 2. You might challenge more capable estimators to try the FOR ESTIMATION EXPERTS section on page 2.

ANSWERS:

1. over 500	2. under 200	3. over $2.00
4. 3000+	5. $3.00-	6. 4000+
7. 6000-	8. 6000-	9. $7.00-
10. $7000-	11. 400+	12. 600-
13. 3000+	14. 200+	
15. 6000-	16. $4.00+	
17. 7250 to 8000-	18. 9000+ to 9500	
19. 4500 to 5000-	20. 550 to 600-	
21. 7000 to 8000	22. 9000+ to 11,000	
23. 16,000+ to 18,000	24. 800 to 900	

1. 440	2. 590	3. 70	4. 3300
5. 3800	6. 1300		

ESTIMATE. TELL HOW YOU THINK.

BE FLEXIBLE. USE A WAY THAT IS QUICK AND EASY TO DO IN YOUR HEAD. MAKE SURE YOUR ESTIMATE IS "IN THE BALLPARK."

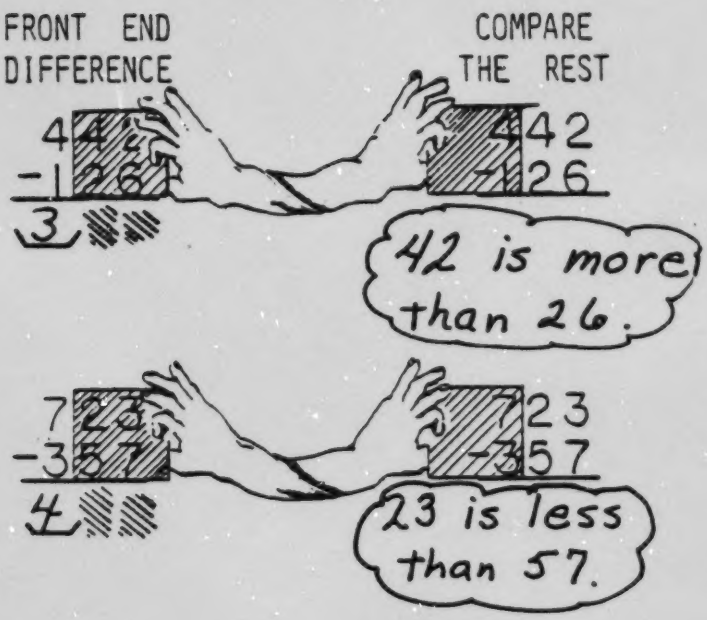
$3.19	$ .10
.84	.57
.45	.58
1.39	.49
.08	.52
2.98	1.20

6-4-TR1

FRONT-END ESTIMATION: SUBTRACTION

SCHOOL	NUMBER OF STUDENTS
PARKER	442
GREENLEAF	126

ABOUT HOW MANY MORE STUDENTS ARE ENROLLED AT PARKER SCHOOL?



TRY THESE:

800	641	$9.31
-239	-419	- 4.69
OVER 600; 600+	OVER 200; 200+	OVER $5.00; $5.00+
OR	OR	OR
UNDER 600; 600-	UNDER 200; 200-	UNDER $5.00; $5.00-

6-4-TR2

FRONT END ESTIMATION: SUBTRACTION

Population Growth In Summerdale.

YEAR	POPULATION
1920	2,321
1940	4,869
1960	6,875
1980	9,327

What was the population increase from 1920 to 1940?

FRONT END DIFFERENCE

4869
−2321

__ thousand

COMPARE THE REST

4869
−2321

869 is more than 321.

FINAL ESTIMATE

_____

From 1960 to 1980?

9327
−6875

__ thousand

9327
−6875

327 is less than 875.

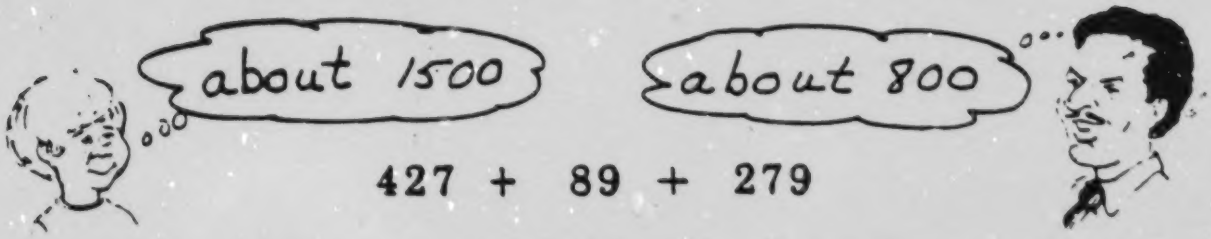
_____

TRY THESE:

	8921 −3545	7000 −5439	4124 −3089
FRONT-END DIFFERENCE	__ thousand	__ thousand	__ thousand
FINAL ESTIMATE	_____	_____	_____

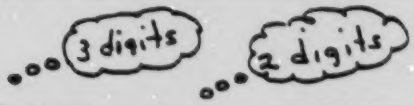
6-4-TR3

WHICH IS THE SENSIBLE ESTIMATE?



427 + 89 + 279

Be careful when there are different numbers of digits!



427 + 89 + 279

(4 + ◌ + 2) hundreds

8367 − 452

(8 − ◌) thousands

Front-end Sum: _____ **Front-end Difference:** _____

Final Estimate: _____ **Final Estimate:** _____

TRY THESE: Tell which digits to add or subtract. Then estimate.

6037 + 249 + 2364

2468 − 391

461 + 47 + 125 + 9

(5387 + 831) − 2290

6-4-TR4

Name_____

Circle the best estimate.

1. 752
- 235
////

over 500
under 500

2. 627
- 486
////

over 200
under 200

3. $3.61
- 1.24
/////

over $2.00
under $2.00

4. 4,621
- 1,264
//////

3,000+
3,000-

5. $5.28
- 2.64
/////

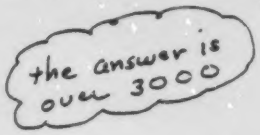
$3.00+
$3.00-

6. 8,624
- 4,187
//////

4,000+
4,000-

Subtract lead digits to estimate.
Write + (the exact answer is more).
Or write - (the exact answer is less).

Ex. 4,627
- 1,529
3,000+



7. $9,624
- 3,810
//////

_____
Estimate

8. 7,694
- 1,872
//////

_____
Estimate

9. $9.17
- 2.86
/////

_____
Estimate

10. $8,111
- 1,888
//////

_____
Estimate

11. 764
- 339
////

_____
Estimate

12. 817
- 284
////

_____
Estimate

13. 4,918 - 1,785 _____ Estimate

14. 485 - 228 _____ Estimate

15. 7,258 - 1,835 _____ Estimate

16. $5.63 - $1.27 _____ Estimate

Name _____

Estimate. Be careful about place value.

Ex. (5 - 0) thousand
5,716 - 335 Estimate: 5,000+

17. 8,125 - 697 _____ Estimate

18. 9,710 - 421 _____ Estimate

19. 5,185 - 371 _____ Estimate

20. 621 - 38 _____ Estimate

Estimate. Be careful about place value. Try to get closer.

Ex. (6 + 0 + 8) thousand
6,643 + 207 + 8,121 14,9000 Estimate

21. 4672 + 834 + 2196 _____ Estimate

22. 792 + 8424 + 1262 _____ Estimate

23. 9126 + 4218 + 3979 _____ Estimate

24. 578 + 8 + 215 + 45 _____ Estimate

FOR ESTIMATION EXPERTS! ! !

↓
621
- 243

ESTIMATE: 400⁻

GETTING CLOSER: 380

Sometimes you may want to get closer with subtraction estimation. Here's how:

1). You know the estimate is under 400 (400⁻) since 21 is less than 43.

2) Look at the tens place. Think: 12 tens - 4 tens = 8 tens. So, a closer estimate is 380.

Try getting closer with these exercises.

1. 763 - 329 Getting Closer _____

2. 931 - 348 Getting Closer _____

3. 642 - 574 Getting Closer _____

4. 6813 - 3561 Getting Closer _____

5. 8692 - 4879 Getting Closer _____

6. 4217 - 2969 Getting Closer _____

NSF ESTIMATION
GRADE 6 - LESSON 5

OBJECTIVE: To estimate sums and differences with larger numbers.

TEACHER BACKGROUND:

With larger numbers the strategies already presented can be used. Selecting "nice" numbers with which to work is particularly helpful, as shown below.

47,249 - 18,087

A. 47,000 - 20,000 is easy to compute, so change 47,249 to 47,000 and 18,087 to 20,000.

Estimate: 27,000

B. 18,000 + 30,000 = 48,000. So change 47,249 to 48,000 and 18,087 to 18,000.

Estimate: 30,000

In presenting the lesson, emphasize:

1) the importance of being flexible when estimating. it is wise to take a few seconds to study the situation to see what might be done to produce a reasonable estimate quickly and mentally.

2) that it is not always easy to adjust an estimate. This is particularly true when the numbers have beer changed to "nice" numbers or when addends have been grouped to "nice" numbers.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 emphasizes examining an answer to see if it is reasonable. For example, in 3013 - 2995, both numbers are about 3000 so the answer shown on the calculator, 1018, does not make sense. Have students look at the numbers as a whole rather than attempting to compute the exact answer.

ANSWERS: The answers to 3013 - 2995 and 2087 + 3652 + 197 are not reasonable.

ESTIMATING WITH LARGER NUMBERS

TR #2: The approach used is front-end. First, the sum of the lead digits is found. Then that number is adjusted up by finding the sum of the digits in the second place from the left. Students may have other ways of doing the adjusting, such as grouping the remaining numbers.

ANSWERS: 230,000 - 240,000 56,000 - 58,000

1,000,000 - 1,100,000

TR #3: *Grouping addends* to "nice" numbers is emphasized. After discussing the sample example, have students suggest other ways the numbers could be grouped.

ANSWERS: $190{,}000^{+} - 210{,}000$ $\qquad$ $140{,}000^{+} - 150{,}000$

$35{,}000 - 42{,}000$

TR #4: Three ways of estimating the difference for a subtraction situation are shown. Discuss each approach and have students suggest other methods.

ANSWERS: $20{,}000 - 25{,}000$ $\qquad$ $45{,}000 - 50{,}000^{-}$

$15{,}000 - 20{,}000^{-}$

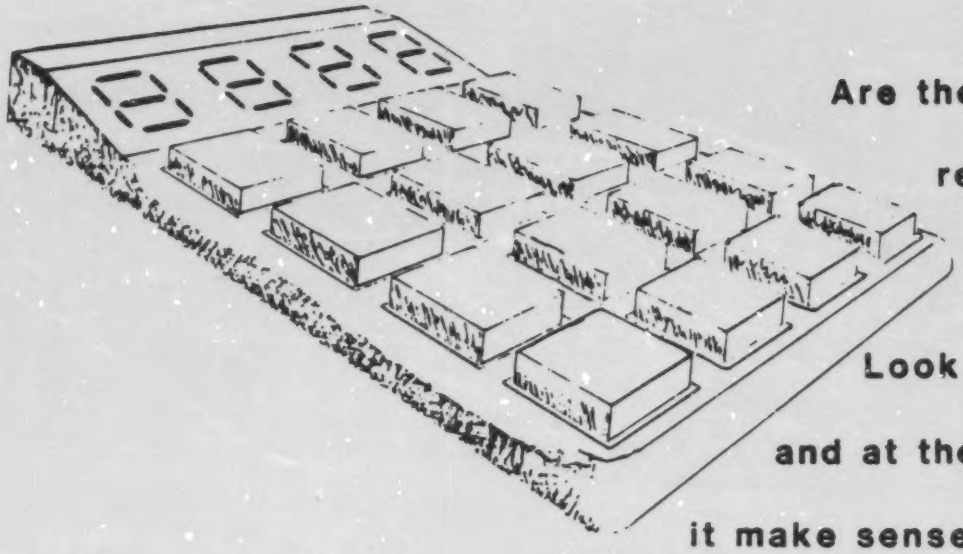
USING THE EXERCISES:

Discuss the directions for Ex. 1-6 and 7-9 before students start their work. Encourage students to be flexible in how they estimate and indicate that "in the ballpark" estimates are acceptable.

ANSWERS:

1. $130{,}000^{+} - 140{,}000$	2. $80{,}000^{+} - 90{,}000$	3. $1{,}300{,}000 - 1{,}500{,}000$
4. $110{,}000^{+} - 122{,}000$	5. $40{,}000^{+} - 50{,}000$	6. $900{,}000^{+} - 1{,}000{,}000$
7. $70{,}000^{+} - 90{,}000$	8. $180{,}000^{+} - 205{,}000$	9. $200{,}000 - 220{,}000$
10. $15{,}000{,}000 - 16{,}500{,}000$	11. $30{,}000{,}000 - 32{,}000{,}000$	12. $50{,}000{,}000 - 58{,}000{,}000$
13. $30{,}000^{+} - 34{,}000$	14. $18{,}000 - 20{,}000^{-}$	15. $400{,}000^{+} - 430{,}000$
16. $40{,}000 - 42{,}000^{-}$	17. $34{,}000 - 40{,}000^{-}$	18. $20{,}000^{+} - 22{,}000$
19. $40{,}000 - 43{,}000$	20. $13{,}500 - 15{,}000$	21. $14{,}000 - 20{,}000^{-}$
22. $25{,}000 - 30{,}000^{-}$	23. $34{,}000 - 40{,}000^{-}$	24. $40{,}000^{+} - 50{,}000^{-}$
25. $50{,}000 - 60{,}000^{-}$	26. $100 - 200^{-}$	27. $200^{+} - 250$
28. $300 - 400^{-}$	29. $20{,}000^{+} - 25{,}000$	

GET YOUR MIND IN GEAR



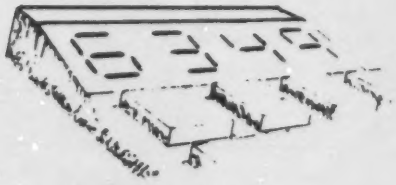
Are these answers reasonable? Use estimation. Look at the problem and at the answer. Does it make sense?

147 + 28 + 32

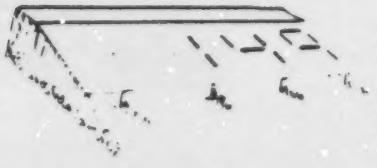
3013 - 2995

2087 + 3652 + 197

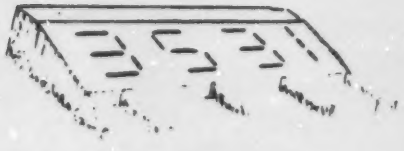
4163 + 2186



346 - 197



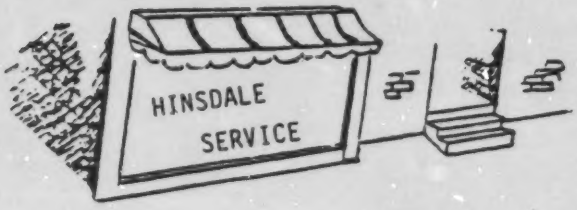
3657 - 126



6-5-TR1

FRONT-END WITH LARGER NUMBERS

ABOUT HOW MUCH DOES MR. HINSDALE SPEND ON SALARIES?



EMPLOYEES	SALARIES
Ms. Schmidt	$52,162
Mr. Walters	$24,921
Mr. Alrich	$32,168
Mrs. Hunt	$18,187

Front-end Sum:

5 Thousand
2 Thousand
3 Thousand
+ 1 Thousand
0 Thousand

110 THOUSAND

Estimate the Rest:

$2 Thousand
$4 Thousand
$2 Thousand
$8 Thousand

16 THOUSAND

Put it Together:

126 THOUSAND OR $126,000

TRY THESE:

36,422	24,826	148,672
81,356	+ 32,924	287,922
24,963		+ 634,127
+ 92,847		

6-5-TR2

GROUPING WITH LARGER NUMBERS

34,126
8,272
65,387
+190,070

Sometimes when you add large numbers, it is easy to group them to NICE numbers.

about 100 — 34 THOUSAND
8 THOUSAND
about 200 — 65 THOUSAND
190 THOUSAND

About 300 thousand
or 300,000.

TRY THESE:

42 THOUSAND
63 THOUSAND
92 THOUSAND
+ 4 THOUSAND

ABOUT: _____

6,827
43,008
+97,634

ABOUT: _____

14,617
2,018
8,152
+15,634

ABOUT: _____

6-5-TR3

USING "NICE NUMBERS" IN SUBTRACTION

CITY	POPULATION
BAINESVILLE	47,249
CARTERVILLE	18,087

About how much larger is the population of Bainesville?

Sandi used front-end.

Hmm... about 29 thousand.
40 thousand – 10 thousand = 30 thousand.
7249 is less than 8087,
so my estimate has to be less than
30 thousand. So... 29 thousand.

Here are some other ways

27,000...
18 thousand is close
to 20 thousand.
47,000 – 20,000 is easy
to do in my head.

... about 30,000.
I thought: 18 thousand
plus what would
make 48 thousand?

TRY THESE:

49,237	61,000	32,879
−26,138	−13,572	−16,127

6-5-TR4

Name _____

Estimate using front-end.

1.	12,673	2.	62,342	3.	267,388
	41,325		1,836		392,430
	+ 85,677		+ 24,267		+ 820,401
	/////////		/////////		/////////
Front-end Estimate:	_____		_____		_____
Getting Closer:	_____		_____		_____

4.	72,147	5.	32,976	6.	162,342
	47,856		14,010		+ 806,978
	+ 1,928		+ 2,827		/////////
	/////////		/////////		
Front-end Estimate:	_____		_____		_____
Getting Closer:	_____		_____		_____

Estimate. Group to nice numbers to work with.

7.	12,677	8.	97,456	9.	162,342
	24,864		24,857		3,940
	+ 48,566		2,738		+ 46,769
	/////////		+ 76,162		/////////
			/////////		
Estimate:	_____		_____		_____

The areas of the seven continents of the world are listed in the chart.

CONTINENT	SQUARE MILES
Asia	16,899,000
Africa	11,506,000
Antarctica	5,500,000
Australia	2,968,000
Europe	3,745,000
North America	9,390,000
South America	6,795,000

10. Estimate the area of North America and South America:

_____ sq. mi.

11. Estimate the area of Europe, Africa and Asia:

_____ sq. mi.

12. Estimate the total land area of the seven continents:

_____ sq. mi.

6-5-p.1

Name _____

Estimate:

13. 68,125 − 34,604 ////////

About _____

14. 27,269 − 8,351 ///////

About _____

15. 667,184 − 249,650 /////////

About _____

16. $45,106 - $3,727 is about _____.

17. $50,000 - $15,325 is about _____.

18. $34,471 - $12,609 is about _____.

19. $85,721 - $42,848 is about _____.

PROFILE OF U.S. HOME BUYERS

	1977	1979	1981
Average Purchase	$44,500	$58,200	$72,750
Average Down Payment	$ 9,000	$12,282	$16,100
Average Monthly Loan Payment	$ 273	$ 401	$ 624

What was the difference in the average cost of a house between:

20. 1977 and 1979? Est: _____

21. 1979 and 1981? Est: _____

22. 1977 and 1981? Est: _____

Home buyers obtain a loan. The loan is for the amount left when the down payment is subtracted from the purchase price. What was the average loan in:

23. 1977? Est: _____

24. 1979? Est: _____

25. 1981? Est: _____

By how much did the average monthly loan payment increase from:

26. 1977 to 1979? Est: _____

27. 1979 to 1981? Est: _____

28. 1977 to 1981? Est: _____

29. In 1981 Mr. and Mrs. Hernandes bought a home for $95,000. About how much more than the average price did they pay? Est: _____

NSF ESTIMATION
GRADE 6 - LESSON 6

OBJECTIVES: To compare rounding and front-end for addition estimation.

To estimate products when one factor is less than 10 using rounding.

To adjust multiplication estimates upwards.

TEACHER BACKGROUND:

In the first part of the lesson rounding and front-end approaches to addition estimation are compared. While front-end estimation has been emphasized, rounding is also a valid approach. However, when there are several addends, as shown in the example at the right, front-end estimation has one major advantage - the numbers being added are visible. When rounding, each rounded number must be held mentally and recalled at the end of the rounding process.

$3.89
4.38
2.63
8.79
+ 5.36

The front-end process also gives estimates that are usually closer to the exact answer because of the adjusting process.

The primary focus of the lesson is estimating for multiplication exercises where one factor is less than 10. This process, in which one factor is rounded may be familiar to students.

6 x 387

a. 387 rounds to 400

b. 6 x 400 = 2400

The lesson extends the process to adjusting the estimate by writing + or - after the initial estimate.

Since 387 is rounded up, the estimate, 2400, is an overestimate. It is greater than 6 x 387. to show this, a "-" sign is written after the estimate.

ESTIMATE: 2400-

Finally, students are shown how to get closer with estimates for situations involving money. For situations such as the one shown at the right it is easy and natural to "get closer" in the manner shown.

3 x $1.19

a. 3 x $1.00 = $3.00

b. $.19 is close to $.20
3 x $0.20 = $0.60

c. ESTIMATE: $3.00 + $0.60
or $3.60

This is an informal introduction to front-end estimation in multiplication which is developed more fully in the seventh grade materials.

6-6-1

You may wish to review two skills that are used in the estimation process. Some students may need additional work with both of them.

1. Multiplying multiples of ten, one hundred or one thousand by a 1-digit number.

6 x 400

a. multiply the lead digits.
   6 x 4 = 24

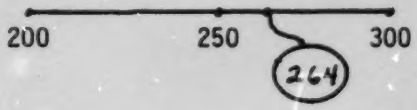
b. write two zeroes 2400

Present several examples written horizontally for students to practice.

3 x 7000 2 x 70 7 x 900
8 x 30 5 x 800 4 x 6000

2. Rounding to hundreds and thousands.

Use the development shown at the right to review the rounding process and the meaning of rounding.



a. 264 is between 200 and 300

b. 264 is closer to 300 since it is more than halfway

c. So, 264 rounds to 300

Present several numbers for students to round.

Round to hundreds.

287 646 592 750

Round to thousands.

1284 3485 6500 4902

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 stresses flexibility in estimation. It is important for students to realize that there are many ways to estimate, and to select the approach appropriate for a given situation. Discuss each of the three methods shown.

1. The first person used front-end estimation.

2. Rounding was used by the second individual. Note that here both methods yield the same estimate.

3. The third person estimated by rounding each number to the next higher dollar. This approach might be used when you want to be sure you have enough money, since it gives an overestimate.

ADDITION ESTIMATION: ROUNDING AND FRONT-END

TR #2: Have each example done using rounding and the front-end strategy. Space is left for recording the rounded numbers. In discussing

the questions at the bottom, bring out that rounding was probably easiest for the middle example since there were only two numbers and they were close to a whole number of dollars. The third example is much easier by front-end estimation since there are four addends.

ANSWERS: 1. Rounding: \$8.00 Front-End: \$8.00^{+} - \$9.25

2. Rounding: \$9.00 Front-End \$9.00^{-}

3. Rounding: \$12.00 Front-End: \$12.00^{+} - \$12.50

MULTIPLICATION ESTIMATION

TR #3: Develop estimation by first rounding one factor. You may need to include additional work on multiplying hundreds and thousands, and rounding. Discuss why the estimate is written as 2400^{-}.

ANSWERS: 1. 1500^{+} 2. 4000^{-} 3. 1400^{+}

4. 24,000^{-} 5. 12,000^{+} 6. 16,000^{-}

TR #4: Here students adjust-up by multiplying the cents. Take time to carefully discuss the two ways of adjusting. In each one \$.19 was rounded to an easy number with which to work. Discuss each TRY THESE exercise with the students.

ANSWERS: 1. \$4.20 - \$4.50

2. \$4.40 - \$4.60

3. \$13.20 - \$14.00

USING THE EXERCISES:

Discuss the directions for each set of exercises before assigning them. Work the same example that precedes Ex. 25-28 with the students.

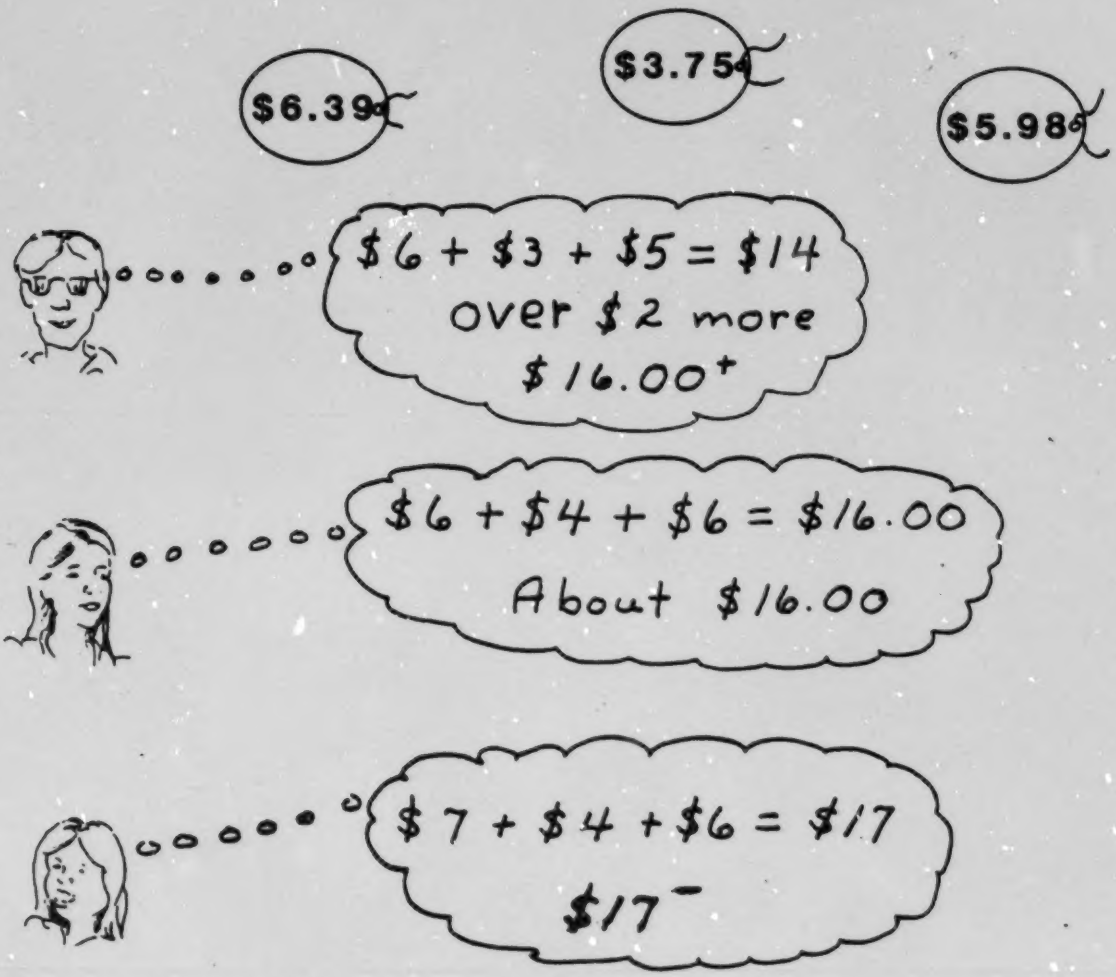
ANSWERS:

1. a. \$13.00	b. \$13.00^{+} - \$14.50	2. a. \$12.00	b. \$12.00 - \$13.00^{-}
3. a. \$11.00	b. \$11.00 - \$11.50	4. a. \$12.00	b. \$11.00 -\$12.00
5. a. 8000	b. 8000^{+} - 9200	6. a. 900	b. 800 - 900
7. 1600	8. 9000	9. 2800	10. 5400
11. 16,000	12. 36,000	13. 2500^{-}	14. 14,000^{+}
15. 240^{+}	16. 6000^{-}	17. 30,000^{-}	18. 400^{-}
19. 1800^{+}	20. 40,000^{-}	21. 27,000^{+}	22. 1600^{-}
23. 320^{-}	24. 30,000^{-}	25. \$6.60	26. \$12.80
27. \$37.50	28. \$7.50		

GET YOUR MIND IN GEAR

There are many ways to estimate.

Tell how each person estimated.



When might you estimate like this?

6-6-TR1

ROUNDING AND FRONT-END

Good estimators use many strategies.
Try these problems two ways.

ROUNDING

$4.29 → ⬭
3.41 → ⬭
+ 1.35 → ⬭

FRONT-END

$4.29
3.41
+1.35

Initial Estimate: _____

Getting Closer: _____

$5.98 → ⬭
+ 2.95 → ⬭

$5.98
+2.95

Initial Estimate: _____

Getting Closer: _____

$4.87 → ⬭
3.29 → ⬭
1.75 → ⬭
+ 2.39 → ⬭

$4.87
3.29
1.75
+2.39

Initial Estimate: _____

Getting Closer: _____

IN WHICH ONE DID ROUNDING WORK BEST?

IN WHICH ONE WAS FRONT-END MUCH EASIER THAN ROUNDING?

6-6-TR2

USE ROUNDING IN MULTIPLICATION

THE SCHOOL PLAY RAN 6 NIGHTS. EACH NIGHT ALL TICKETS WERE SOLD. THE THEATER SEATS 387 PEOPLE. HOW MANY TICKETS WERE SOLD?

6 x 387

6 x 300 → About 1800

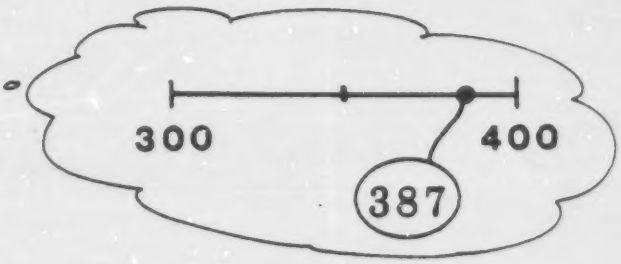
6 x 400 → About 2400

Both 1800 and 2400 are in the ballpark. But

- 387 IS CLOSER TO 400

AND

- 6 X 400 IS EASY TO DO



So, 2400 is a better estimate.

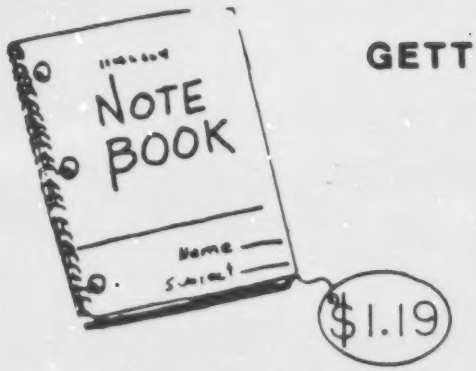
You can adjust the estimate: 2400 −

TRY THESE: **Round and give the closer estimate.**

3 X 542	8 X 467	7 X 219
4 X 5816	6 X 2482	4 X 3762

6-6-TR3

GETTING CLOSER IN MULTIPLICATION



3 pads of paper would be .... $3.00+
3 × $1.00 = $3.00

Sometimes it's easy to get closer.

Use $1.20

$3.00+
Adjust Up:
3 × $.20
Estimate: $3.60

Use $1.25

$3.00+
Adjust Up:
3 quarters
Estimate: $3.75

Now you try getting closer.

	CLOSE ENOUGH	GETTING CLOSER
$1.49 each Buy 3	$ 3.00+	_____
$2.29 each Buy 2	$ 4.00+	_____
$3.38 each Buy 4	$12.00+	_____

6-6-TR4

Name _____

Solve each example using rounding. Then do it using front-end with adjusting. Put a √ next to the way that worked best for the example.

1. $7.38
   2.44
   + 4.29
   //////
   Rounding: _____
   Front-End: _____

2. $8.97
   + 2.99
   /////
   Rounding: _____
   Front-End: _____

3. $2.08
   3.96
   + 5.12
   /////
   Rounding: _____
   Front-End: _____

4. $3.27
   2.63
   1.38
   + 4.75
   /////
   Rounding: _____
   Front-End: _____

5. 6,298 + 426 + 2,349
   Rounding: _____
   Front-End: _____

6. 346 + 51 + 8 + 482
   Rounding: _____
   Front-End: _____

Estimate. (400) (3000)

7. 4 x 372 _____ Estimate

8. 3 x 2897 _____ Estimate

9. 7 x 432 _____ Estimate

10. 9 x 633 _____ Estimate

11. 8 x 2179 _____ Estimate

12. 4 x 9209 _____ Estimate

Write + or - in the ○.

3 x 247 is greater than 600. So, write a + to show that the exact answer is more than 600.

Initial Estimate: 600

Adjusted Estimate: 600 ⊕

13. 5 x 489 — 2500 ○ Estimate

14. 7 x 2165 — 14,000 ○ Estimate

15. 4 x 63 — 240 ○ Estimate

16. 6 x 998 — 6,000 ○ Estimate

17. 5 x 5701 — 30,000 ○ Estimate

18. 8 x 49 — 400 ○ Estimate

Name _____

Estimate by rounding one factor then adjust.

Ex. 7 X 279 2100 Estimate

19. 3 X 639 _____ Estimate

20. 5 X 7730 _____ Estimate

21. 9 X 3102 _____ Estimate

22. 2 X 780 _____ Estimate

23. 8 X 39 _____ Estimate

24. 6 X 4792 _____ Estimate

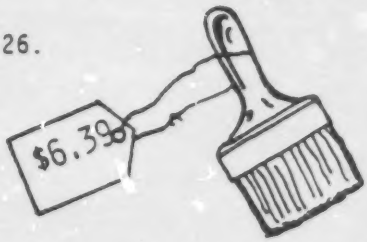
Try getting closer with money. Use dimes, quarters and half dollars to adjust.

4 X $2.47

1. Use the front digit. 4 X $2 = 8
2. Work with the cents. 47¢ is closer to 50¢
   4 X 50¢ = $2
3. Write the estimate. $8 + $2 = $10.00 Estimate

25.

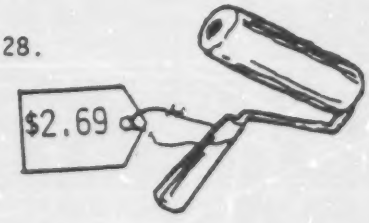
Front-End (3 X $2)
Adjust Up (3 X 20¢)
Estimate _____

26. 

Front-End (2 X $6)
Adjust Up (2 X 40¢)
Estimate _____

27.

Front-End (5 X $7)
Adjust Up (5 X 50¢)
Estimate _____

28. 

Front-End (3 X $2)
Adjust Up (3 X 50¢)
Estimate _____

NSF ESTIMATION
GRADE 6 - LESSON 7

OBJECTIVES: To estimate products by rounding both factors.

To estimate products by rounding one factor to 10, 100, or 1000.

To adjust estimates.

TEACHER BACKGROUND:

When both factors have at least two digits it is usually most effective to round each factor to the lead digit and find the product of the rounded numbers.

71 x 238

ESTIMATE: 14,000

A variation occurs when one factor is close to 10, 100 or 1000. Here one factor is rounded and the other factor is multiplied by 10, 100 or 1000. This strategy produces a closer estimate than rounding both factors.

645 x 94

ESTIMATE: 64,500

Students need to be skillful in multiplying multiples of ten, hundred, thousand, etc., and multiplying a number by 10, 100 or 1000. These skills have not been mastered by many sixth grade students and need additional attention. Thus, you may need to spend additional time on these prerequisites. One important aspect of developing the first skill is highlighting the steps that are used to find the product, as shown below.

40 x 600

1. Multiply the lead digits. 40 x 600 = 24 ___
2. Then write as many zeros as there are in both factors. 40 x 600 = 24,000

The lesson also encourages students to adjust estimates when possible. Four examples are shown below.

a. 28 x 56	ESTIMATE:	1800^-	Both factors are rounded up. So 1800 is an overestimate.
b. 62 x 23	ESTIMATE:	1200^+	Both factors are rounded down. So 1200 is an underestimate.
c. 37 x 96	ESTIMATE:	3700^-	37 x 100 is greater than 37 x 96 So, 3700 is an overestimate.
d. 62 x 79	ESTIMATE:	4800	When one factor is rounded up and the other factor is rounded down, there is no general rule that can be used for adjusting.

One additional variation is presented in the lesson. When both factors are close to the midpoint, a more precise estimate can be obtained by rounding one factor up and the other factor down.

36 x 75 ESTIMATE: 2800 Use 40 x 70

or

ESTIMATE: 2400 Use 30 x 80

TEACHING THE LESSON

GET YOUR MIND IN GEAR

TR #1 helps students recognize when an estimate is sufficient. In discussing each question, you might also bring out that an estimate is useful as a check on exact computation.

ANSWERS: 1. No; No; Yes; Probably yes

2. Yes; No 3. No; Yes

ESTIMATING PRODUCTS

TR #2: Begin by reviewing multiplying multiples of ten (40 x 60). Then extend the work to other examples (40 x 300, 600 x 500).

Develop the work shown on the transparency. Record the rounded numbers in the bubbles to help students remember the numbers they must multiply.

ANSWERS:			
	2400	32,000	1600
	5600	30,000	100,000
	4500	8,000	1400

TR #3: Review multiplying numbers by 10, 100 and 1000. Then develop rounding one factor to 10, 100 or 1000. Have students tell why the estimate is written as 3700^- (37 x 100 is greater than 37 x 96. So, 3700 is an overestimate and the answer to 37 x 96 is <u>less than</u> 3700.)

You might compare rounding both factors to rounding one factor to show how the latter produces a closer estimate.

37 x 96 = 3552 ESTIMATE 1: 40 x 100 = 4000

ESTIMATE 2: 37 x 100 = 3700

Point out that both estimates are acceptable.

ANSWERS:			
	6500^-	4700^+	$32,000^-$
	2400^-	450^+	$3,200^-$
	$43,600^-$	2700^-	$60,000^-$
	$\$240^+$	$\$8200^-$	$\$4000^-$

TR #4: Develop the first three examples on adjusting estimates carefully, making sure that students understand the reasoning. Then develop the fourth example on getting a closer estimate. Discuss the TRY THESE exercises with students.

ANSWERS: 1500 or 1600 1200$^-$ 1800$^+$ or 2100

1600 2400 or 2100 1200

USING THE EXERCISES

On exercises 16-27 students are not required to adjust the estimates, although they may do so if they wish. This is treated in exercises 10-15. The THINK IT THROUGH exercises at the bottom of page 2 are not an integral part of the lesson. They are designed to help students think about the size of an answer. These should be handled as "bonus" exercises.

ANSWERS:

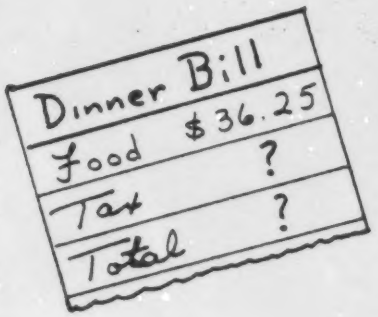
1. 2800	2. 4800	3. 12,000
4. 48,000	5. 450,000	6. 490,000
7. 3600	8. 2700	9. 800
10. 18,000	11. 9000	12. 45,000
13. 5600	14. 30,000	15. 300,000
16. 6300	17. 12,000	18. 180,000
19. 36,000	20. 4900	21. 1800
22. 27,000 (26,700)	23. 8000	24. 150,000
25. 120,000	26. 3500	27. 15,000
28. 7400$^-$	29. 2800$^+$	30. \$170$^+$
31. 5000$^-$	32. \$6700$^-$	33. 580$^+$
34. 53,000-	35. 47,500$^-$	36. 8600$^+$
37. under	38. hard to tell	39. over
40. hard to tell	41. over	42. under

43. 2400; 780-800; 1600(-); 1600; 1600; 2400(-)

THINK IT THROUGH

1. 3 or 4 digits (10 x 10 = 100 and 99 x 99 = 9801)
2. 2 or 3 digits
3. 1,2 or 3 digits (also 0 digits)

GET YOUR MIND IN GEAR



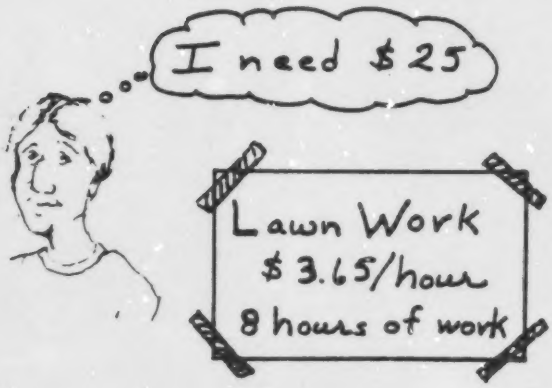
Is an estimate enough when:

THE WAITRESS FIGURES 5% TAX?

THE WAITRESS FINDS THE TOTAL?

THE CUSTOMER FIGURES A 15% TIP?

THE CUSTOMER CHECKS THE BILL?



Is an estimate enough when:

SHAWN DECIDES IF HE'LL MAKE ENOUGH MONEY?

THE BOSS FIGURES OUT HOW MUCH TO PAY SHAWN?

Is an estimate enough when:

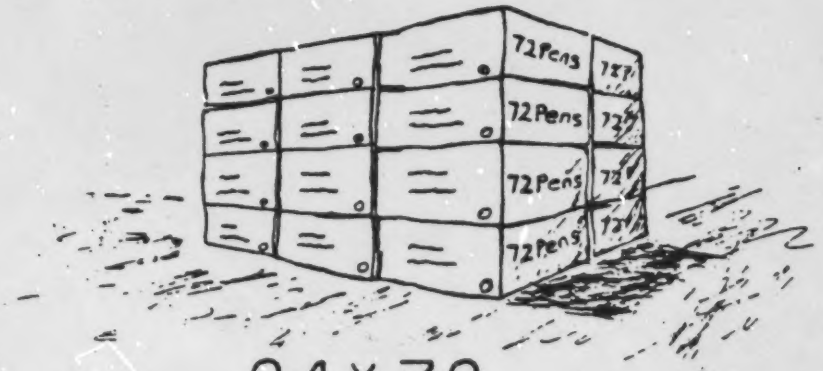
THE ACCOUNTANT FIGURES OUT HOW MUCH MONEY WAS MADE ON TICKET SALES?

THE NEWSPAPER REPORTS HOW MANY PEOPLE ATTENDED THE GAME?

6-7-TR1

ESTIMATING PRODUCTS

KARL'S BOOKSTORE ORDERED 24 BOXES OF PENS. ABOUT HOW MANY PENS IS THIS?



24 × 72

20 70

① **Round both factors:** 24 × 72

② **Multiply the rounded numbers:** 20 × 70 = 1400

Now try these!

42 × 63

384 × 79

37 × 42

67 × 83

512 × 62

223 × 489

92 × 51

41 × 206

24 × 67

6-7-TR2

USE 10, 100 OR 1000

When one factor is close to 10, 100 or 1000... it's easy to just change that factor and multiply.

100

37 X 96

Estimate: 3700

Since I rounded 96 up, I have to adjust down.

TRY THESE:

100	100	1000
65 x 98	47 x 103	989 x 32
24 x 99	45 x 11	97 x 32
98 x 436	99 x 27	60 x 896
24 x $10.12	82 x $99.63	4 x $986

6-7-TR3

ADJUSTING ESTIMATES

28 x 56

Round Up

30 x 60

1800 is an overestimate. So, I'll adjust down: 1800⁻

62 X 23

Round Down

60 x 20

1200 is an underestimate. So, I'll adjust up: 1200⁺

62 X 79

Down x **Up**

60 x 80

You can't really tell. So, I'll just say... 4800.

36 X 75

Since they are both close to the middle, I'll round one up and one down: 2800.

Up 40 x **Down** 70

TRY THESE

35 X 45	**27 X 38**	**92 X 21**
42 X 37	**64 X 35**	**42 X 27**

6-7-TR4

Name _____

Write the products.

1. 40 X 70 = _____ 2. 80 X 60 = _____ 3. 300 X 40 = _____

4. 80 X 600 = _____ 5. 500 X 900 = _____ 6. 700 X 700 = _____

Circle the best estimate.

7. 57 X 65 360 3600 36,000	8. 92 X 28 270 2700 27,000	9. 41 X 22 800 8000 80,000
10. 634 X 27 180 1800 18,000	11. 32 X 329 900 9000 90,000	12. 92 X 496 450 4500 45,000
13. 65 X 84 560 5600 56,000	14. 328 X 121 300 3000 30,000	15. 479 X 608 3000 30,000 300,000

Estimate: Round each factor and multiply.

16. 92 X 67 (90, 70) Estimate: _____

17. 287 X 36 Estimate: _____

18. 567 X 323 Estimate: _____

19. 437 X 86 Estimate: _____

20. 72 X 69 Estimate: _____

21. 59 X 26 Estimate: _____

22. 94 X 267 Estimate: _____

23. 369 X 18 Estimate: _____

24. 312 X 465 Estimate: _____

25. 269 X 423 Estimate: _____

26. 71 X 48 Estimate: _____

27. 502 X 27 Estimate: _____

Name _____

Estimate. Look for numbers close to 10, 100, 1000, etc.
Adjust your estimate by putting a - or +.

Example: 15 x 9 Est. 150⁻

28. 74 x 98 _____ 29. 107 x 28 _____ 30. $11.14 x 17 _____

31. 5 x 987 _____ 32. $98.72 x 67 _____ 33. 58 x 12 _____

34. 895 x 53 _____ 35. 475 x 96 _____ 36. 105 x 86 _____

Circle the choice that best describes the Estimate.

37. 32 x 42 1200
over under hard to tell

38. 24 x 86 1800
over under hard to tell

39. 57 x 78 4800
over under hard to tell

40. 72 x 28 2100
over under hard to tell

41. 56 x 48 3000
over under hard to tell

42. 92 x 61 5400
over under hard to tell

43. The automatic dishwasher at the Star Inn can wash 78 dishes at one time. Estimate the number of dishes washed each day.

	LOADS	DISHES WASHED
Sunday	32	
Monday	13	
Tuesday	21	

	LOADS	DISHES WASHED
Thursday	19	
Friday	23	
Saturday	28	

Circle the answers. There is more than one answer for each one.

1. When you multiply 2 two-digit numbers, there can be ? digits in the product. 1 2 3 4
2. When you add 2 two-digit numbers, there can be ? digits in the sum. 1 2 3 4
3. When you subtract 2 three-digit numbers, there can be ? digits in the difference. 1 2 3 4

6-7-p.2

NSF ESTIMATION
GRADE 6 - LESSON 8

OBJECTIVES: To find the size of the quotient.

To estimate quotients with 1-digit divisors.

TEACHER BACKGROUND:

This is the first of four lessons on estimation in division. There are two aspects of division estimation presented in Lesson 8:

1) Find the size of the quotient. Students find the location of the first quotient digit, which tells how many digits there are in the quotient. This indicates whether the quotient is in the ones, tens, hundreds, etc. In the example at the right there are 3 digits in the quotient which means the quotient must be in the hundreds. So, 450 is in the right range.

$6\overline{)2875}$

$6\overline{)2874}$

$6\overline{)2875}$

Finding the size of the quotient is a form of estimation and also helps students recognize whether a quotient is sensible.

45 (450) 4500

2) Estimate the quotient using the first quotient digit. Many students are not aware that when they find the first quotient digit they have an acceptable estimate. The 4 in the quotient (see example) indicates that the quotient is between 400 and 500, or in the 400's. This approach does not always give the closest number of hundreds, but provides a reasonable estimate.

$6\overline{)2875}$ (4 in the hundreds)

ESTIMATE: 400

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 serves two purposes. First, it continues the emphasis on recognizing sensible answers, and second, it provides an informal introduction to the content of the lesson. Have students scan the examples to find the ones that do not have sensible answers. You might have each example read aloud to help the focus on the numbers (357 divided by 4, etc.). Discourage them from trying to work the problems. Have students tell why the problems they identify don't have sensible answers.

ANSWERS: The answers to $3\overline{)456}$, $8\overline{)6560}$ and $7\overline{)2476}$ are not sensible. In these cases the quotient does not have the correct number of digits.

FIND THE SIZE OF THE QUOTIENT

TR #2: Show the top of the transparency and indicate that they are going to find whether the quotient has 2 digits (in the tens), 3-digits (in the hundreds) or 4-digits (in the thousands).

Then present the steps. In step 2 point out that 1 thousand 8 hundred is 18 hundred and they are thinking of the number of hundreds in all. There are enough hundreds to divide (split) into 3 equal parts.

For step 3 stress that they know 1) the quotient is in the hundreds and 2) there are 3 digits in the quotient. Write 60, 600 and 6000 and have students tell which is the most reasonable estimate.

Go back to TR #1 and have students see whether the number of digits in the quotient is correct.

ANSWERS: 4 digits (in the thousands)

3 digits (in the hundreds)

3 digits (in the hundreds)

3 digits (in the hundreds)

3 digits (in the hundreds)

2 digits (in the tens)

TR #3: Present the problem at the top and the first step. State that when they know the size of the answer (in the tens) they have an estimate.

Develop step 2. Indicate that carrying out the first step of long division provides a refinement of the rough estimate. Now they know that the quotient is <u>in the thirties</u>. (While this process does not find the closest number of tens, 30 is still a good estimate.)

ANSWERS: hundreds, 400 thousands, 4000

hundreds, 100 tens, 30

TR #4: Here students get an opportunity to apply estimation skills.

ANSWERS: $40; $200; $3; $.40; $400

<u>USING THE EXERCISES</u>:

Do the sample exercise at the top of page 1 before assigning the exercises. The THINK IT THROUGH exercises are an extension of the lesson and can be assigned to more capable students.

ANSWERS:

1. hundreds	2. tens	3. hundreds	
4. hundreds	5. tens	6. hundreds	
7. thousands	8. thousands	9. tens	
10. 60	11. 1000	12. 100	
13. $0.60	14. $8.00	15. $10.00	
Out-in-Left-Field:	#1, #4, #6		
16. 500	17. 30 (40)	18. 2000	19. 400
20. 2000	21. 2000	22. 70	23. 10,000
24. $3.00	25. $0.60	26. $7.00	27. $1.00
28. 30	29. 1000	30. 300	31. 200 (300)
32. $20.00	33. $100		

THINK IT THROUGH

1. greater than	2. less than
3. less than	4. less than

GET YOUR MIND IN GEAR

IN THE BALLPARK?

OR

OUT IN LEFT FIELD?

See if you can quickly spot the answers that are "out in left field." They don't make sense . Do not try to work the problems again.

NAME Carlos H.

1. $4\overline{)357}$ 89 r 1

2. $3\overline{)452}$ 15 r 3

3. $6\overline{)2472}$ 412

4. $8\overline{)6560}$ 82

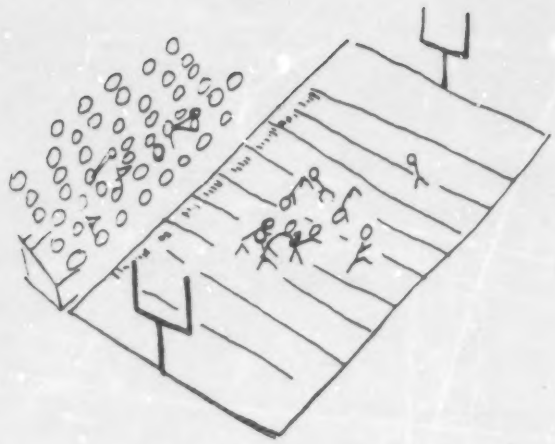
5. $7\overline{)2476}$ 3537 r 1

6. $44\overline{)3282}$ 74 r 26

Good estimators should recognize reasonable answers!

6-8-TR1

FIND THE SIZE OF THE QUOTIENT



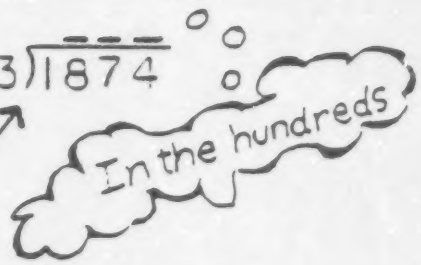
1874 PEOPLE ATTENDED THE FIRST 3 FOOTBALL GAMES. WHAT WAS THE AVERAGE ATTENDANCE?

$3\overline{)1874}$

① Are there enough thousands to divide? $3\overline{)1374}$

② Are there enough hundreds in all to divide? $3\overline{)1874}$

So the quotient has 3 digits. $3\overline{)1874}$



To be sensible the quotient must be in the hundreds.

TRY THESE: Tell the size of each quotient

$6\overline{)8781}$ $4\overline{)2370}$ $7\overline{)901}$

$3\overline{)673}$ $5\overline{)4572}$ $8\overline{)465}$

6-8-TR2

ESTIMATE THE QUOTIENT

MR. ROBINSON DROVE HIS COMPACT CAR 252 MILES ON 8 GALLONS OF GASOLINE. ABOUT HOW MANY MILES PER GALLON DID HE AVERAGE?

$8\overline{)252}$

1

What is the size of the quotient?

THE QUOTIENT HAS _____ DIGITS.

THE QUOTIENT IS IN THE _____S.

2

Get a closer estimate by finding the first digit of the quotient.

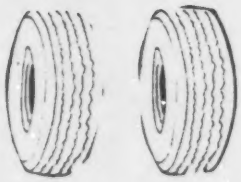
Think $8\overline{)25}$ (3)
So, $8\overline{)252}$ is about 3 tens or 30

Now try these!

$6\overline{)2567}$	$3\overline{)12,372}$	$5\overline{)603}$	$9\overline{)284}$
IN THE_____	IN THE_____	IN THE_____	IN THE_____
ABOUT_____	ABOUT_____	ABOUT_____	ABOUT_____

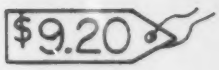
6-8-TR3

ABOUT HOW MUCH FOR ONE?

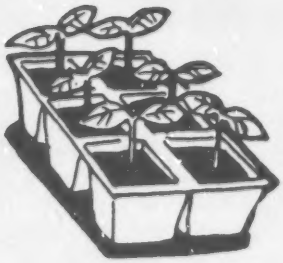


TIRES
4 for $179 $4 $40 $400

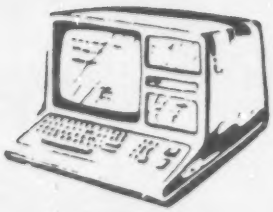
CRUISE
6 days for $1250 $2 $20 $200



Split 3 ways $0.03 $.30 $3.00



6 for $2.43 $0.04 $.40 $4.00



$2579
6 months to pay $40 $400 $4000

6-8-TR4

Name _____

Make marks to show the number of digits in each quotient. Then tell if the quotient is in the tens, hundreds or thousands.

Ex. $6\overline{)928}$ in the tens

1. $9\overline{)1882}$ in the _____

2. $6\overline{)164}$ in the _____

3. $8\overline{)4682}$ in the _____

4. $3\overline{)827}$ in the _____

5. $7\overline{)524}$ in the _____

6. $5\overline{)4207}$ in the _____

7. $4\overline{)5300}$ in the _____

8. $8\overline{)9806}$ in the _____

9. $2\overline{)162}$ in the _____

Choose the best estimate. Think about the size of the quotient.

10. $3\overline{)195}$

6 60 600

11. $5\overline{)6300}$

10 100 1000

12. $8\overline{)945}$

1 10 100

13. $4\overline{)\$2.42}$

\$0.06 \$0.60 \$6.00

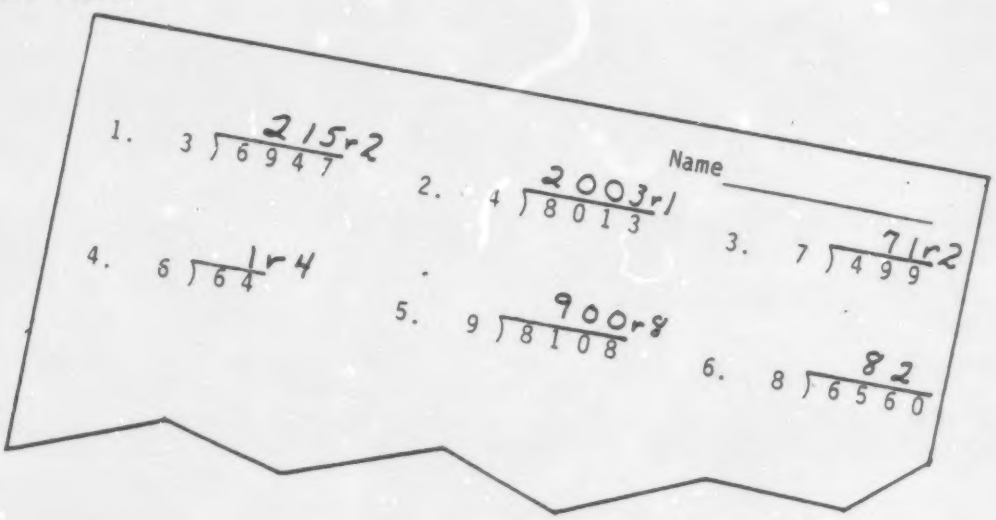
14. $5\overline{)\$43.20}$

\$0.80 \$8.00 \$80.00

15. $6\overline{)\$95.00}$

\$0.10 \$1.00 \$10.00

Put an X by the answers that are "out in left field."



Name_____

Estimate by finding the first digit in the quotient. Ex. $3\overline{)251}$ Estimate: 80

16. $6\overline{)3240}$ Est:_____

17. $8\overline{)306}$ Est:_____

18. $3\overline{)7346}$ Est:_____

19. $2\overline{)849}$ Est:_____

20. $7\overline{)15,213}$ Est:_____

21. 9000 ÷ 4 Est:_____

22. $5\overline{)371}$ Est:_____

23. 97,012 ÷ 9 Est:_____

24. \$6.47 ÷ 2 Est:_____

25. $3\overline{)\$1.97}$ Est:_____

26. \$42.12 ÷ 6 Est:_____

27. $4\overline{)\$4.81}$ Est:_____

28. There are 291 students in one school and 9 classrooms. If each teacher has about the same number of students, about how many students are there in a room?

_____ students

29. An athlete can eat 3250 calories a day. If he eats 3 meals a day, what is the average number of calories per meal?

_____ calories

30. The Mississippi River is 2470 miles long. The Longworth family traveled along the river for 7 days. About how many miles did they average a day?

_____ miles

31. Wilson Middle School students put on a play. There were 3 performances. If 863 tickets were sold, what was the average attendance at each performance?

_____ people

32. Mr. Gonzales bought 4 peach trees for \$82.64. About what was the cost of each tree?

_____ a tree

33. The Anderson family bought 6 new dining room chairs for \$710. About how much did each chair cost?

_____ a chair

Write greater than or less than.

1. 1600 ÷ 4 = 400, so 1621 ÷ 4 is _____ 400.
2. 25 x 25 = 625, so 24 x 25 is _____ 625.
3. 24,000 ÷ 3 = 8000, so 23,575 ÷ 3 is _____ 8000.
4. 1000 - 500 = 500, so 2000 - 525 is _____ 500.

NSF ESTIMATION
GRADE 6 - LESSON 9

OBJECTIVE: To estimate quotients using compatible numbers.

TEACHER BACKGROUND:

Now students estimate by finding the closer number of tens, hundreds or thousands in the quotient. To do this students need to recognize which multiple of the divisor the part being divided is closer to. As shown at the right, 26 is close to 27, which is a multiple of 3. Thus, the closer estimate is 90.

$3\overline{)262}$

$3\overline{)27}$

ESTIMATE: 90

Numbers such as 3 and 27, where one is a multiple of the other, are referred to as compatible numbers. Students may need practice in this and future lessons in recognizing multiples of a number.

The use of compatible numbers is not limited to rounding up, as shown by the example at the right. The work is similar to finding the first quotient digit (Lesson 8), although the thinking may differ slightly.

$8\overline{)4112}$

$8\overline{)40}$

ESTIMATE: 500

Two further notes:

1. Some students may have difficulty with finding which multiple of the divisor the number being divided is closer to. Do not expect all students to master compatible numbers at this time. They are used again in Lessons 10 and 11. Also, exercises have been selected in which the closer multiple is more easily seen (that is, the use of $7\overline{)268}$ rather than $7\overline{)248}$).

2. In this lesson students should find the closer estimate. In other situations accept both the lower and upper estimates. For $6\overline{)2852}$, 400 and 500 should both be considered acceptable.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

The target game (TR #1) encourages decision-making in estimation. One of the six pairs of factors will be close to the target number. One strategy is to eliminate pairs which are obviously not possible. For the first situation 426 x 32 is obviously too large, while 19 x 32 is too small.

ANSWERS: 62 and 19 9 and 41

8 and 59 32 and 79

41 and 49 26 and 42

USING COMPATIBLE NUMBERS TO ESTIMATE

TR #2: Put a table like the one at the right on the board and have students identify multiples of 7. Circle the multiples of 7.

MULTIPLES OF 7						
42		63				25
				14		
	27					
57			17			43
34				49		
		56			28	52
21			30			

Develop the two approaches to the problem at the top of TR #2. In the second method point out that 35 is a multiple of 5 and 34 is close to 35. Introduce the term "Compatible Numbers." Indicate that $70 is a closer estimate because 344 is closer to 350 than 300.

Do the TRY THESE exercises with the students.

ANSWERS: 700 50 60

90 700 60

TR #3: Review the two steps in estimating presented at the top. Develop the first TRY THESE exercise with the class. Then have them try the six exercises at the bottom before discussing them with the class.

ANSWERS: 4000 600 900

30 3000 20

TR #4: Here students have a chance to apply their estimation skills.

ANSWERS: $20 $30 $90

$400 $300 $3

USING THE EXERCISES:

Discuss the first exercise in each set with the students before having them begin to work independently.

ANSWERS:

1. 24, 18, 48, 42, 54, 60 2. 27, 36, 54, 63, 72, 81

3. $4\overline{)28}$ 4. $9\overline{)36}$ 5. $6\overline{)42}$

6. $7\overline{)28}$ 7. $5\overline{)30}$ 8. $8\overline{)24}$

9. $6\overline{)360}$ 10. $4\overline{)3600}$ 11. $7\overline{)4200}$

12. $8\overline{)3200}$ 13. $5\overline{)3000}$ 14. $9\overline{)5400}$

15. 500 16. 600 17. 600 18. 500

19. 800 20. 700 21. 700 22. 800

23. 300 24. 300 25. 300

26. 90 27. 2000 28. $2.00

29. 500 30. $70.00 31. 7000

32. 700 33. 800 34. 800

35. 900 36. 600 37. 600

38. $0.20 39. $0.30 ($0.33)

40. $0.40 41. $0.40 42. $0.90

43. $0.80 44. pencil case

45. binder notebook

GET YOUR MIND IN GEAR

Choose the two numbers whose product is closest to the target number.

426 62 79 32

9 41 31 122

8 59 117 42

32 624 41 79

22 49 41 11

26 19 42 5

6-9-TR1

USING COMPATIBLE NUMBERS TO ESTIMATE

WHAT IS THE APPROXIMATE MONTHLY PAYMENT?

In the Ballpark:

$5\overline{)344}$ In the tens... about $60

Getting Closer:

34 is close to 35 and 35 can be divided by 5.

$5\overline{)35}$ About $70

5 AND 35 ARE COMPATIBLE NUMBERS SINCE 5 DIVIDES 35 EVENLY.

WHY IS $70 A CLOSER ESTIMATE?

TRY THESE:

Use compatible numbers to find the closer estimate.

$6\overline{)41}$ $6\overline{)36}$ $6\overline{)42}$

$7\overline{)34}$ $7\overline{)28}$ $7\overline{)35}$

$5\overline{)31}$ $5\overline{)30}$ $5\overline{)35}$

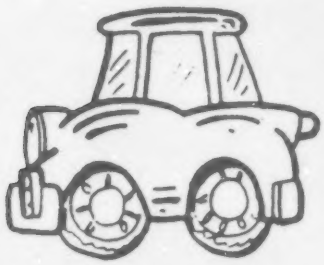
$3\overline{)26}$ $3\overline{)24}$ $3\overline{)27}$

$8\overline{)58}$ $8\overline{)56}$ $8\overline{)64}$

$9\overline{)52}$ $9\overline{)45}$ $9\overline{)54}$

6-9-TR2

MORE ABOUT COMPATIBLES



$4025
6 monthly payments
? per month

Estimate

FIND THE SIZE OF THE QUOTIENT:

$6\overline{)4025}$ ∴ In the hundreds

USE COMPATIBLE NUMBERS:

$6\overline{)42}$ HUNDRED ∴ 7 hundred

40 is close to 42. 6 and 42 are compatible.

ABOUT $700 PER MONTH

∴ $8\overline{)32}$ hundred → 4 hundred

TRY THESE:

Which is closer?

$8\overline{)3568}$ ∴ $8\overline{)40}$ hundred → 5 hundred

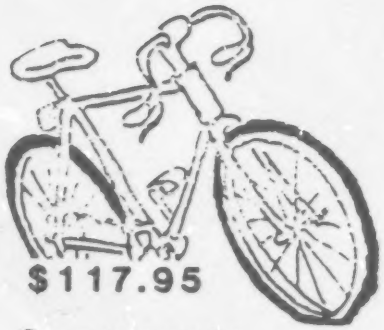
Estimate. Use compatible numbers.

$4\overline{)17,972}$ $7\overline{)4136}$ $5\overline{)4398}$

$8\overline{)234}$ $9\overline{)26,658}$ $7\overline{)128}$

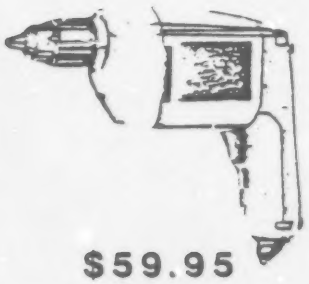
6-9-TR3

ABOUT HOW MUCH IS EACH PAYMENT?



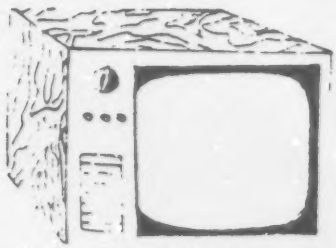
$117.95

6 payments



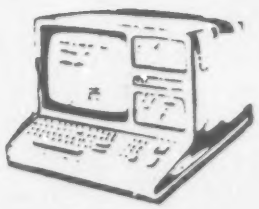
$59.95

2 payments



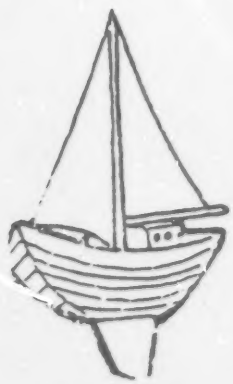
$519.95

6 payments



$2599

9 payments



$2165

5 payments

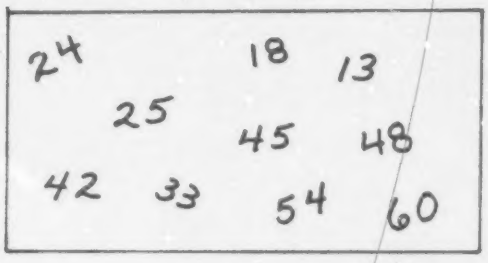
$14.95

5 payments

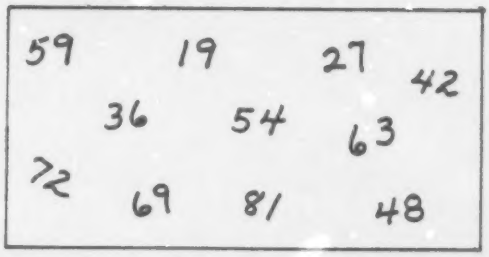
6-9-TR4

Name _____

Circle multiples of 6.



Circle multiples of 9.



Change the dividend so the numbers are compatible. Find the closer multiple.

Ex. $3\overline{)17} \rightarrow 3\overline{)18}$

3. $4\overline{)27} \rightarrow 4\overline{)\ \ }$

4. $9\overline{)35} \rightarrow 9\overline{)\ \ }$

5. $6\overline{)43} \rightarrow 6\overline{)\ \ }$

6. $7\overline{)27} \rightarrow 7\overline{)\ \ }$

7. $5\overline{)31} \rightarrow 5\overline{)\ \ }$

8. $8\overline{)23} \rightarrow 8\overline{)\ \ }$

Circle the example that gives the closest estimate.
Think of compatible numbers.

9. $6\overline{)356}$

$6\overline{)300}$ $6\overline{)360}$

10. $4\overline{)3527}$

$4\overline{)3200}$ $4\overline{)3600}$

11. $7\overline{)4106}$

$7\overline{)4200}$ $7\overline{)4900}$

12. $8\overline{)3099}$

$8\overline{)2400}$ $8\overline{)3200}$

13. $5\overline{)3197}$

$5\overline{)3000}$ $5\overline{)3500}$

14. $9\overline{)5362}$

$9\overline{)4500}$ $9\overline{)5400}$

Choose the closest estimate for each example.

4000 ÷ 8 = 500 4800 ÷ 8 = 600	15. 4169 ÷ 8 : _____ 17. 4702 ÷ 8 : _____	16. 4699 ÷ 8 : _____ 18. 4302 ÷ 8 : _____
4900 ÷ 7 = 700 5600 ÷ 7 = 800	19. 5425 ÷ 7 : _____ 21. 5099 ÷ 7 : _____	20. 5201 ÷ 7 : _____ 22. 5550 ÷ 7 : _____

6-9-p.1

Name _____

Write the closest estimate in the oval. Use compatible numbers.

23. $5\overline{)1420}$ ( )

24. $3\overline{)861}$ ( )

25. $7\overline{)1962}$ ( )

26. $529 \div 6$ ( )

27. $8\overline{)18{,}261}$ ( )

28. $4\overline{)\$7.28}$ ( )

29. $9\overline{)4635}$ ( )

30. $\$139.51 \div 2$ ( )

31. $54{,}111 \div 8$ ( )

The total amount of mail received by the Charles Company during its 5-day work week is shown. Find the average number of pieces per day.

	TOTAL MAIL	DAILY AVERAGE
32. Week 1	3463	
34. Week 2	4216	
36. Week 3	2901	

	TOTAL MAIL	DAILY AVERAGE
33. Week 4	3927	
35. Week 5	4467	
37. Week 6	3199	

Estimate to the nearest dime the cost for one.
(You can get closer if it looks easy.)

38. 8 markers $1.49

_____

39. 3 pens $1.00

_____

40. 4 pencil boxes $1.75

_____

41. 6 pencil cases $2.29

_____

42. 7 spiral notebooks $6.15

_____

43. 5 binder notebooks $3.85

_____

Circle the better buy.

44. pencil box

or

pencil case

45. spiral notebook

or

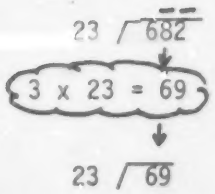
binder notebook

NSF ESTIMATION
GRADE 6 - LESSON 10

OBJECTIVE: To estimate quotients using compatible numbers when dividing by a 2-digit number.

TEACHER BACKGROUND:

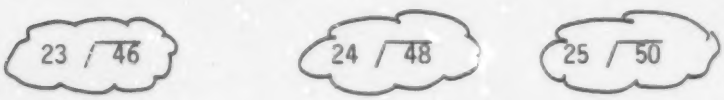
Work with compatible numbers is extended to 2-digit divisors. Here the whole divisor is used as shown in the example at the right. In the example, one thinks of multiples of the divisor and adjusts the dividend. $23 \overline{)68}$ is changed to $23 \overline{)69}$ since 69 is a multiple of 23 ($3 \times 23 = 69$).



ESTIMATE: 30

Both the dividend and divisor can be changed as shown in the examples below.

$23 \overline{)476}$



Compatible numbers is a valuable estimation tool for many division examples. It requires the ability to recognize pairs of numbers that are multiples and works best when the quotient is 2, 3, 4 or 5. You can expect a variety of ways of finding compatible numbers and this is reflected in the exercises. Again, do not expect all students to develop a high level of mastery. Exposure to the approach is valuable and most students will be able to use it for some examples. Lesson 11 presents other ways to estimate for 2-digit divisors.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 provides practice with addition and multiplication estimation. Let students discuss different ways in which the estimating could be done and how they might get closer with their estimates.

ANSWERS:			
	18 watermelons	$36 - $40	(a little less than $36)
	36 lemonades	$36-	
	Total cost:	$70 - $80	

USING COMPATIBLE NUMBERS TO ESTIMATE

TR #2: Present the top part and review estimating the size of the quotient. Then show the various ways of using compatible numbers in step 2. Stress that the whole divisor is used. Show step 3 where the estimate is given.

TR #3: Students are provided with practice in making and recognizing compatible numbers. At the top have students mentally double, triple and quadruple each number. Fill in the numbers as they are given. Point out that each number is a multiple of the first number in each column.

At the bottom practice is provided in recognizing pairs of compatible numbers and in changing numbers so that they are compatible. For the examples that must be changed, there are several ways of doing them.

TR #4: Present the work at the top as a summary of the lesson. Then have students estimate the monthly payment using compatible numbers. To encourage the use of compatible numbers, the exercises have not been written in the usual long division format ($\overline{)\quad}$).

ANSWERS: $30 $30 $80 $200 $200

USING THE EXERCISES:

Work the first exercise in each set with the students before assigning the pages so they know what to do (Ex. 1, 5, 11, 17, 26).

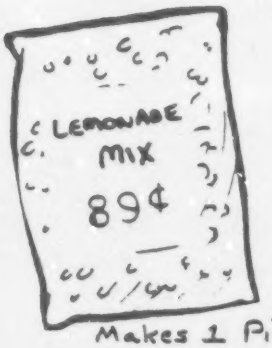
ANSWERS:

1. 40/120	2. 25/75	3. 35/70	4. 15/45
42/126	23/69	34/68	13/39
5. 70/420	6. 61/366	7. 70/210	
8. 81/243	9. 25/125	10. 50/200	
11. 20	12. 200	13. 20	
14. 200	15. 400	16. 900	
17. 4	18. 400	19. 500	
20. 30	21. 30 or 40	22. 600 or 700	
23. 200	24. 800	25. 300	
26. 300	27. 40 or 50	28. 200	
29. 70	30. 50	31. 100	

GET YOUR MIND IN GEAR

Tell How You Think

THE SCHOOL GAVE ONE WATERMELON AND TWO PITCHERS OF LEMONADE TO EACH CLASS. ABOUT HOW MUCH DID THE SCHOOL SPEND?



Makes 1 Pitcher

$1.98
EACH

18 CLASSES

- HOW MANY WATERMELONS? _____

  ABOUT HOW MUCH? _____

- HOW MANY LEMONADES? _____

  ABOUT HOW MUCH? _____

- PUT IT TOGETHER: ESTIMATE _____

How did you think? How else could you think?

6-10-TR1

ESTIMATE WITH 2-DIGIT DIVISORS

A CAR WENT 476 MILES ON 23 GALLONS OF GAS. ABOUT HOW MANY MILES PER GALLON IS THAT?

1. Find the size of the quotient.

$23\overline{)47\not{6}}$

ARE THERE ENOUGH TENS IN ALL TO DIVIDE?

$23\overline{)476}$ ... in the tens

THE QUOTIENT HAS TWO DIGITS.

2. Use compatible numbers. Use the whole divisor.

3. Put it Together.

$\overset{2\rightarrow 20}{23\overline{)476}}$

About 20 miles per gallon.

6-10-TR2

MAKE AND FIND COMPATIBLE NUMBERS

	12	42	31	14	23	13
DOUBLE	___	___	___	___	___	___
TRIPLE	___	___	___	___	___	___
QUADRUPLE	___	___	___	___	___	___

TRY THESE. Are the numbers compatible?
If not, change the example so the numbers are compatible.

Compatible numbers divide evenly!

$24\overline{)48}$	$32\overline{)63}$	$11\overline{)21}$
$14\overline{)25}$	$23\overline{)69}$	$42\overline{)126}$
$33\overline{)64}$	$13\overline{)67}$	$21\overline{)40}$
$12\overline{)36}$	$41\overline{)160}$	$13\overline{)26}$
$31\overline{)91}$	$22\overline{)42}$	$34\overline{)68}$

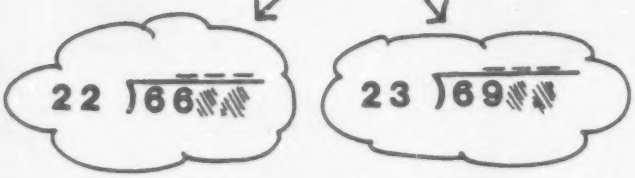
6-10-TR3

PRACTICE ESTIMATING

To make compatible numbers you can either:

Change One Number

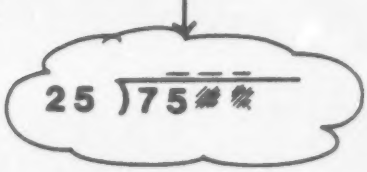
$22\overline{)6972}$



ABOUT _____

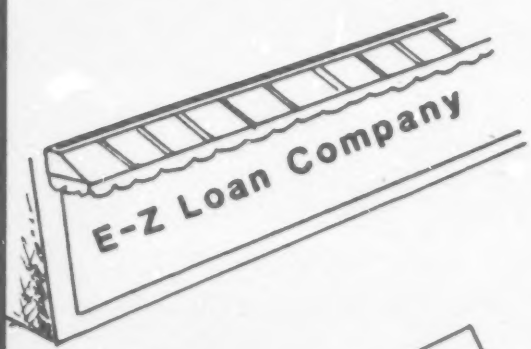
Change Both Numbers

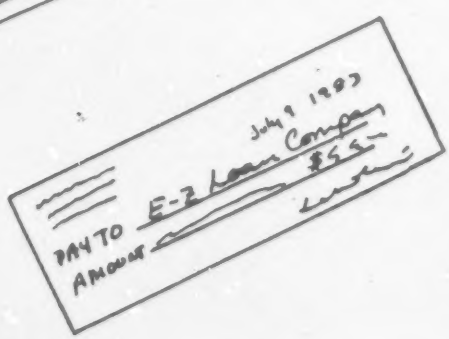
$22\overline{)6972}$



ABOUT _____

TRY THESE: Estimate the monthly payments.





Amount	Time	Monthly Payment
$375	12 MONTHS	_____
$726	24 MONTHS	_____
$1562	18 MONTHS	_____
$7150	36 MONTHS	_____
$9500	48 MONTHS	_____

6-10-TR4

Name _____

Put an X on the examples that use compatible numbers.

1. 42/1362

40/120

42/126

40/140

2. 23/7016

25/75

23/70

23/69

3. 34/7263

35/70

34/68

30/72

4. 13/42638

13/40

15/45

13/39

Circle the example that uses compatible numbers.

5. 72/439

70/420 70/439

6. 61/372

61/366 61/400

7. 73/20162

70/201 70/210

8. 81/23782

81/243 80/200

9. 26/11563

25/125 25/115

10. 54/2263

50/200 54/230

Write an example that uses compatible numbers. Then estimate.

11. 24 / 4 9 3

Estimate: _____

12. 82/ 1 8 6 7 2

Estimate: _____

13. 61 / 1 1 9 6

Estimate: _____

14. 44 / 8 5 9 7

Estimate: _____

15. 32 / 1 2 6 7 8

Estimate: _____

16. 39 / 3 6 3 0 9

Estimate: _____

Name_____

Write marks to show the number of digits in the quotient. Then estimate.

17. $58\overline{)236}$ Estimate:_____

18. $68\overline{)27,806}$ Estimate:_____

19. $42\overline{)19,635}$ Estimate:_____

20. $89\overline{)2636}$ Estimate:_____

21. $29\overline{)1108}$ Estimate:_____

22. $78\overline{)46,672}$ Estimate:_____

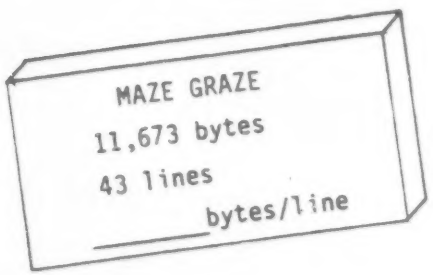
23. $32\overline{)5982}$ Estimate:_____

24. $61\overline{)47,292}$ Estimate:_____

25. $23\overline{)6547}$ Estimate:_____

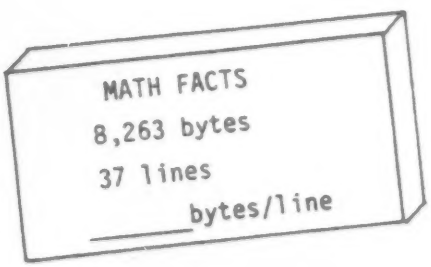
Estimate the number of bytes per line for these computer games.

26.



27.

28.



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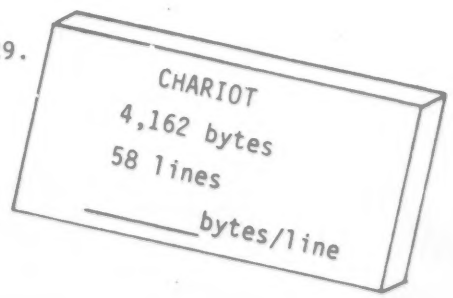
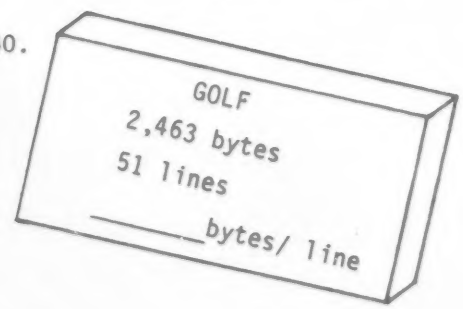
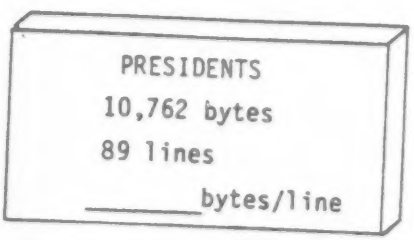


30.



31.



NSF ESTIMATION
GRADES 6 - LESSON 11

OBJECTIVES: To estimate quotients by rounding the divisor when dividing by a 2-digit number.

TEACHER BACKGROUND:

The use of compatible numbers may not seem appropriate for some examples. This lesson presents two ways of estimating that begin by rounding the divisor to the nearest ten.

1. Find the first quotient digit. This parallels the work in Lesson 8. After rounding the divisor the dividend can be truncated (rather than rounded) and the first digit of the quotient is found. While this may not give the closer number of tens, hundreds,thousands, etc. it produces a reasonable estimate in an efficient way.

$63\overline{)4375}$

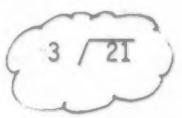
a. $60\overline{)4375}$

round to tens ↖ ↑ truncate

b. $6\overline{)4375}$ → 7 → 70

2. Use compatible numbers. After rounding the divisor and truncating the dividend, students look for compatible numbers to get the closest number of tens, hundreds, thousands, etc. for the estimate.

$30\overline{)2016}$ → $30\overline{)2016}$ → 7 → 70



Throughout all work with division estimation, encourage students to first indicate the number of digits in the quotient, so their estimate will have the correct place value.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 emphasizes flexibility in estimation through examining different ways of estimating an answer for a problem. Display the first line (Be a Flexible Estimator!) and have students suggest what they think being flexible means. Then display the three points and discuss them with students.

Present each of the examples without showing the approaches used. Have students find an estimate and describe what they did. Then show and discuss the methods displayed on the transparency.

ESTIMATE BY ROUNDING THE DIVISOR

TR #2: Display the top half of the transparency. Have students suggest how they might estimate the quotient. Then show the step for rounding the divisor. Point out that even though 6 $\overline{)44}$ was used to find the first quotient figure, the estimate is 70 not 700, since the quotient has two digits.

ANSWERS: (5 $\overline{)36}$) 70 (4 $\overline{)9}$) 20

TR #3: Present the title and the example. Ask students how compatible numbers can be used to get the closest estimate. Then show how the two students thought and have them tell why Patsy's estimate was better (29 is closer to 30 than to 25, so 60 is a closer estimate).

ANSWERS: 50 60 6

6 100 7

SUMMARY

TR #4: The work on the top two-thirds of the page emphasizes the importance of finding the number of digits in the quotient so the estimate has the correct place value. Present the problem at the top and have students tell why the estimate is incorrect. Then review how they can find the number of digits in the quotient.

The work at the bottom compares using the whole divisor and compatible numbers methods, and the two methods where the divisor is first rounded to tens. Bring out that both approaches are useful and that it is important to study the example to see which is the best way to estimate for that example.

ANSWERS: 1. 20, 10 2. 30, 60

USING THE EXERCISES:

Do the first exercise in each set with the students (#1, 10, 16, 22, 34) so they understand what they are to do. The exercises have been constructed to enable students to experience success in estimating.

ANSWERS:

1. tens 2. hundreds 3. hundreds

4. hundreds 5. tens 6. thousands

7. tens 8. tens 9. hundreds

10. 10 11. 5000 12. 100 13. 2000 14. 20 15. 300

16. 48 17. 35 18. 24 19. 36 20. 49 21. 12

22. 2000 23. 7 or 8 24. 200 25. 30 26. 5 or 6 27. 600

28. 20 29. 800 or 900 30. 200 31. 50 or 60

32. 30 33. 6 or 7 34. 20^- 35. 30^- 36. 20^+ 37. 10^+

38. 40^+

YOUR MIND IN GEAR

Be a Flexible Estimator!

Look for short cuts.

Look for ways of Getting Closer.

Look for easy numbers to work with.

How would you estimate for these?

26 x 45

Both are near the middle. I'll round one up and one down.
30 x 40 = 1200

25 x 40 is easy!
25 x 4 = 100
So 25 x 40 = 1000
1000+

8,364
5,348
+ 28,677

I'll use 30 thousand for 28 thousand.
8 and 5 is 13.
So... about 43 thousand

28 + 5 = 33
and 8 more is 41. So....
about 41,000

50,862
− 24,841

25 and 25 is 50.
So, my estimate is a little over 25,000.

6-11-TR1

DIVISION ESTIMATION: ROUND THE DIVISOR

THE P.T.E. ORGANIZATION HAS 4450 FLYERS TO DISTRIBUTE. 63 PEOPLE HAVE VOLUNTEERED TO HELP. ABOUT HOW MANY FLYERS SHOULD EACH PERSON GET?

$63\overline{)4450}$

Sometimes compatible numbers don't seem easy to use.

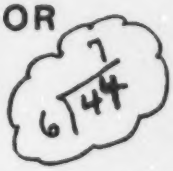
$63\overline{)445}$

60

$63\overline{)4450}$

SO... ROUND THE DIVISOR

and



Find the first digit of the quotient. Write it in the tens place.

$$63\overline{)4450}$$
$$\quad\ 7$$

About 70 per person

TRY THESE

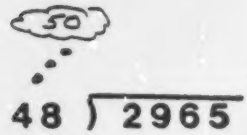
ROUND THE DIVISOR

$47\overline{)3657}$

$38\overline{)963}$

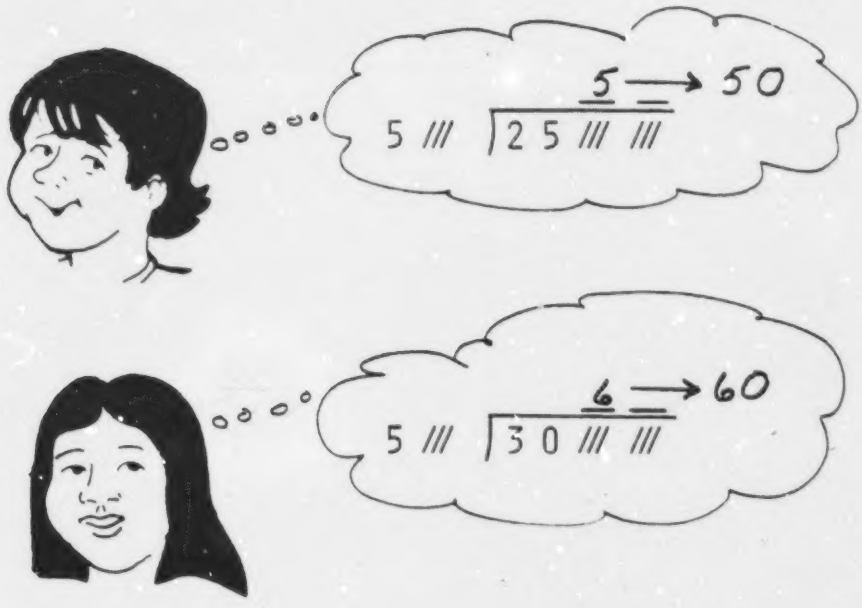
6-11-TR2

ROUND THE DIVISOR



and

USE COMPATIBLE NUMBERS



Tell how each person thought.

Why is Patsy's estimate better?

TRY THESE

ROUND THE DIVISOR - USE COMPATIBLE NUMBERS TO ESTIMATE

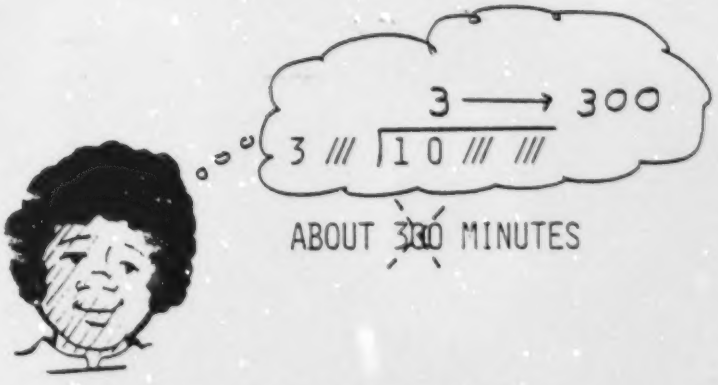
72 ⟌ 3417 29 ⟌ 1939 37 ⟌ 237

18 ⟌ 117 53 ⟌ 5181 47 ⟌ 362

6-11-TR3

PULL YOUR THINKING TOGETHER

A JET USES 32 GALLONS OF FUEL PER MINUTE. AT THIS RATE, HOW LONG CAN IT FLY ON 1000 GALLONS?



Find Carl's mistake.

Find the number of digits in the quotient first.

32) 1000

CAN YOU DIVIDE:

THOUSANDS BY 32? 32) 1000

HUNDREDS BY 32? 32) 100

TENS BY 32? 32) 10

THE QUOTIENT IS IN THE TENS.

THINK HOW YOU CAN ESTIMATE

1. USE THE WHOLE DIVISOR.

35) 713 ⋯ 35 | 70

18) 197 ⋯ 18 | 180

2. ROUND THE DIVISOR. USE THE FIRST DIGIT.

49) 1625 ⋯ 5 | 16

72) 4087 ⋯ 7 | 42

6-11-TR4

NAME _____

Make marks to show the number of digits in each quotient. Then write whether the quotient is in the tens, hundreds, or thousands.

1. 42 / 867 — in the _____

2. 27 / 19672 — in the _____

3. 48 / 9267 — in the _____

4. 71 / 31682 — in the _____

5. 92 / 3927 — in the _____

6. 16 / 24931 — in the _____

7. 43 / 1236 — in the _____

8. 58 / 4215 — in the _____

9. 47 / 8592 — in the _____

Choose the best estimate. Think about the size of the quotient.

10. 32/427

10 100 1,000

11. 18/91,063

50 500 5,000

12. 27/3,210

10 100 1,000

13. 41/82,637

20 200 2,000

14. 63/1,236

20 200 2,000

15. 62/18,627

30 300 3,000

Change each dividend to a compatible number.

16. 63 / 4791 — 6 / ___

17. 49 / 3467 — 5 / ___

18. 82 / 2236 — 8 / ___

19. 57 / 3567 — 6 / ___

20. 71 / 478 — 7 / ___

21. 42 / 11263 — 4 / ___

NAME _____

Estimate. Find the first quotient digit or use compatible numbers.

22. $42\overline{)84,261}$ (?)

Estimate: _____

23. $51\overline{)392}$ (?)

Estimate: _____

24. $38\overline{)8,293}$ (?)

Estimate: _____

25. $28\overline{)927}$ (?)

Estimate: _____

26. $32\overline{)178}$ (?)

Estimate: _____

27. $48\overline{)32,652}$ (?)

Estimate: _____

28. $18\overline{)425}$ (?)

Estimate: _____

29. $92\overline{)80,962}$ (?)

Estimate: _____

30. $28\overline{)6,283}$ (?)

Estimate: _____

31. $41\overline{)2,363}$ (?)

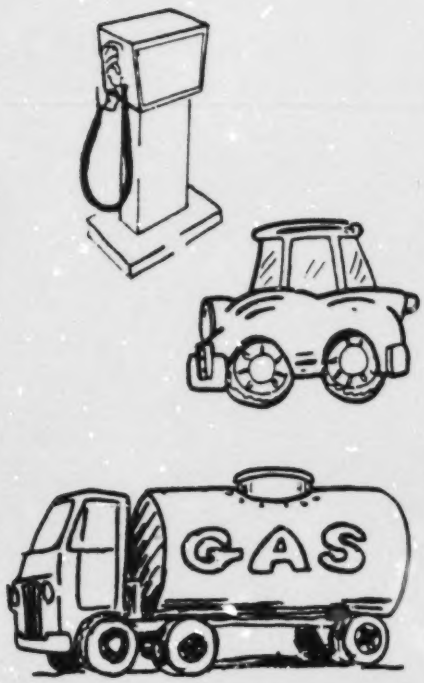
Estimate: _____

32. $39\overline{)1,263}$ (?)

Estimate: _____

33. $58\overline{)417}$ (?)

Estimate: _____



Choose the better estimate for the average miles traveled on each gallon of gasoline.

	MILES TRAVELED	GALLONS	ESTIMATES	
34.	233	13	20^+	20^-
35.	372	14	30^+	30^-
36.	124	5	20^+	20^-
37.	400	28	10^+	10^-
38.	337	8	40^+	40^-

NSF ESTIMATION
GRADE 6 - LESSON 12

OBJECTIVES: To identify the part of a region that is shaded.

To identify fractions that are close to 1 or $1/2$.

To use the fact that $1/2 + 1/2 = 1$ to determine whether the sum of two fractions is greater or less than 1.

TEACHER BACKGROUND:

The major focus of the lesson is to recognize fractions that are close to 1, $1/2$, or 0. This is important in being able to estimate the sum of fractions and mixed numbers, which is presented in Lesson 13. This work also encourages students to think about the amounts represented by fractions. The idea is easy for students to learn and contributes to their understanding of fractions.

Fractions Close to 1

$9/10$	$17/20$	$13/12$
$4/5$		$6/7$

A fraction is close to 1 when the numerator and denominator are about the same size.

Fractions Close to $1/2$

	$7/15$	
$5/8$		$5/12$
$4/10$	$13/24$	

A fraction is close to $1/2$ when the denominator is about twice as large as the numerator.

Fractions Close to 0

$4/30$	$1/6$	$3/19$
$3/17$		$1/8$

A fraction is close to 0 when the numerator is very small in comparison to the denominator.

One can also reccgnize when a fraction is greater than $1/2$ or less than $1/2$.

$3/8$ is less than $1/2$ since 3 is less than $1/2$ of 8

$4/9$ is less than $1/2$ since 4 is less than $1/2$ of 9 (or 9 is more than twice as large as 4).

This idea is applied in constructing fractions whose sum is greater or less than 1.

$1/2 + 1/2 = 1$. So $3/9 + 3/8$ is less than 1.

$1/2 + 1/2 = 1$. So $1/2 + 5/8$ is greater than 1, since $5/8$ is greater than $1/2$.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 again addresses the theme of recognizing sensible answers. Here students are encouraged to examine the number of digits in the answer to see if the size of the answer is reasonable. Discuss each of the situations at the top of the page with students.

1.	32,456 ÷ 81	3 digits	The first digit in the quotient should be written above the 4. There are not ten-thousands or thousands to divide, but there are enough hundreds (324 hundreds ÷ 81)
2.	6382 + 8437	5 digits	The sum of two 4-digit numbers will have 4 or 5 digits. Since the sum of the thousands is greater than 10, the sum will have 5 digits.
3.	21 x 89	4 digits	The product of two 2-digit numbers will have 3 or 4 digits. Since 20 x 80 = 1600, the product will have 4 digits.
4.	4368 - 3949	3 digits	The difference must have 4 or fewer digits. Looking at the first two digits in each number (43 hundreds - 39 hundreds) shows that the answer is less than 1000, but in the hundreds.

Now have students identify which of the examples at the bottom have sensible answers, based on the number of digits in the answer. The problems that do not have sensible answers are:

6440 ÷ 8 4267 + 649 84 x 57 427 - 36 667 + 288 8154 ÷ 27

RECOGNIZING THE AMOUNTS REPRESENTED BY FRACTIONS

TR #2: In the first problem, have students look at the shaded bar. Then have them look at the three fractions and determine which fraction fits the amount shown. Put in the lines to show the 8 equal parts. Since one part is shaded, the diagram shows 1-eighth. You might also have a student come and shade $^1/_2$ and $^1/_4$ of the bar. Proceed in a similar manner for the next three examples.

Then have students shade the approximate amount of the bar for each fraction. If any student has difficulty, point out that for $^4/_9$, the 9 indicates that there are 9 equal parts and the 4 indicates that 4 of the 9 parts are shaded.

TR #3: Develop each of the 3 situations. Bring out through discussion:

a. that a fraction is close to 1 when the numerator and denominator are about the same size.

b. that a fraction is close to 0 when the numerator is very small in comparison to the denominator.

c. that a fraction is close to $^1/_2$ when the denominator is about twice as large as the numerator (or, when the numerator is about half as large as the denominator).

Then have students select the fractions at the bottom that are close to 1, 0, and $^1/_2$.

ANSWERS: Close to 1: $^3/_4$, $^4/_5$, $^7/_9$

Close to 0: $^3/_{17}$, $^1/_8$, $^2/_{13}$

Close to $^1/_2$: $^7/_{13}$, $^4/_9$, $^6/_{11}$

To summarize this portion of the lesson have students suggest several fractions that are close to 1, close to 0 or close to $^1/_2$.

TR #4: Before using transparency 4, write $^3/_8$, $^4/_9$, $^5/_{12}$ and $^5/_{11}$ on the board. Have students tell you whether they are all less than $^1/_2$ or greater than $^1/_2$. Bring out that in each case the numerator is less than half the denominator (or the denominator is more than twice the numerator) so they are less than $^1/_2$.

Repeat for $^5/_8$, $^5/_9$, $^7/_{12}$, and $^6/_{11}$, pointing out that they are all greater than $^1/_2$.

Now show the equation at the top of the transparency and the next portion. Tell students that they are to choose two fractions from those in the box that will make the sum greater than 1 when added to $^1/_2$. Since $^5/_8$ and $^7/_8$ are greater than $^1/_2$, $^1/_2 + ^5/_8$ and $^1/_2 + ^7/_8$ are greater than 1.

Now have them choose the two fractions that will make the sum less than 1 when added to $^1/_2$.

TRY THESE: Draw students' attention to the three exercises under Greater than 1. In each case they are to complete the second fraction so the sum of the two fractions is greater than 1. Remind them that they know that $^1/_2 + ^1/_2 = 1$. Repeat for the three exercises under Less than 1.

USING THE EXERCISES:

The exercises on page 1 are the key ones for the lesson and all students should be able to do them independently. You may wish to do the exercises on page 2 with the students or do the first one in each set with them.

ANSWERS:

1. $^2/_{13}$ 2. $^7/_8$ 3. $^6/_{11}$ 4. $^3/_{18}$

9. $^1/_{10}$, $^1/_9$, $^3/_{20}$, 10. $^7/_{12}$, $^7/_{15}$, $^6/_{13}$ 11. $^{17}/_{15}$, $^9/_{11}$, $^{14}/_{17}$

12-14: Answers will vary

15. a. $3/11$ or $5/11$ b. $7/11$ or $9/11$

16. a. $3/8$ b. $5/8$, $6/8$, or $7/8$

17. a. $3/16$ or $5/16$ b. $11/16$

18. a. $3/20$ or $7/20$ b. $13/20$ or $17/20$

19. $1/2 + 3/5$ $9/16 + 5/8$

20. less than $1/2$ 21. more than $1/2$

22. more than $1/2$ 23. less than $1/2$

YOUR MIND IN GEAR

Recognize Sensible Answers

How many digits are in the answer? How do you know?

32,456 ÷ 81	____	DIGITS
6382 + 8437	____	DIGITS
21 x 89	____	DIGITS
4368 - 3949	____	DIGITS

Look at the number of digits to see if the answer is sensible.

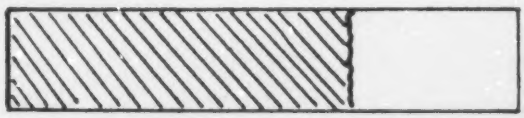
6440 ÷ 8 = 85	627 - 519 = 108
667 + 288 = 1355	4267 + 649 = 10,757
84 x 57 = 42,588	8154 ÷ 27 = 32
427 - 36 = 67	37 x 84 = 3108

6-12-TR1

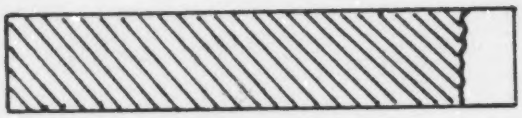
ESTIMATING FRACTIONAL PARTS

CHOOSE THE FRACTION THAT SHOWS <u>ABOUT</u> HOW MUCH IS SHADED.

$\frac{1}{8}$ $\frac{1}{4}$ $\frac{1}{2}$



$\frac{1}{3}$ $\frac{3}{10}$ $\frac{2}{3}$



$\frac{9}{10}$ $\frac{2}{5}$ $\frac{5}{8}$

$\frac{1}{6}$ $\frac{2}{5}$ $\frac{11}{12}$

NOW, YOU DO THE SHADING.

$\frac{4}{9}$

$\frac{2}{10}$

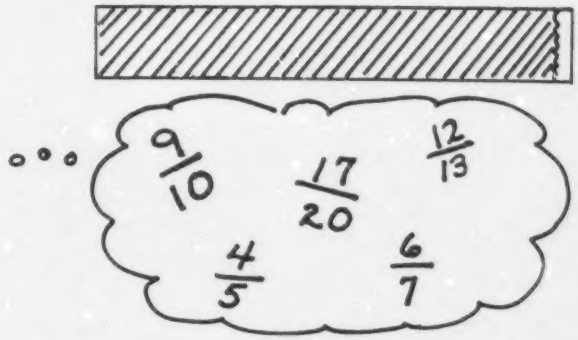
$\frac{7}{8}$

6-12-TR2

FRACTIONS CLOSE TO 1, 0, AND $\frac{1}{2}$

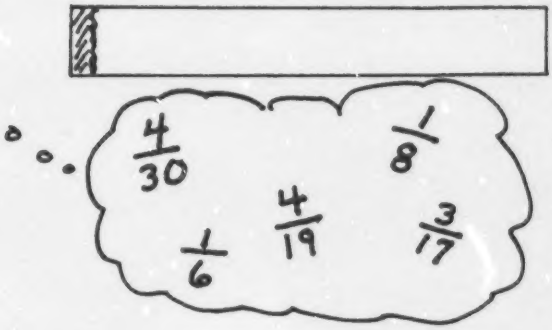
Close to 1

HOW CAN YOU TELL WHEN A FRACTION IS CLOSE TO 1?



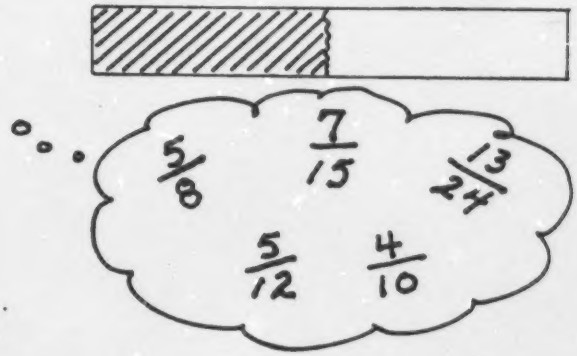
Close to 0

HOW CAN YOU TELL WHEN A FRACTION IS CLOSE TO 0?



Close to $\frac{1}{2}$

HOW CAN YOU TELL WHEN A FRACTION IS CLOSE TO $\frac{1}{2}$?

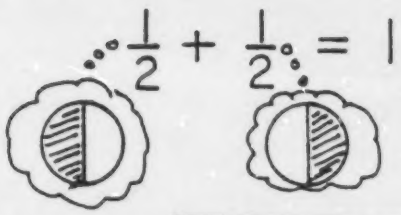


TRY THESE

CLOSE TO 1 ?	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{7}{12}$	$\frac{4}{5}$	$\frac{7}{9}$
CLOSE TO 0 ?	$\frac{3}{5}$	$\frac{3}{17}$	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{2}{13}$
CLOSE TO $\frac{1}{2}$?	$\frac{7}{13}$	$\frac{2}{7}$	$\frac{4}{9}$	$\frac{6}{11}$	$\frac{8}{9}$

6-12-TR3

SUMS NEAR ONE



$\frac{1}{2}$ + ?

$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$

Choose two fractions to make the sum:

GREATER THAN 1: ___ AND ___

LESS THAN 1: ___ AND ___

TRY THESE

Greater than 1

$\frac{1}{2} + \frac{\square}{20}$

$\frac{1}{2} + \frac{\square}{9}$

$\frac{1}{2} + \frac{\square}{12}$

Less Than 1

$\frac{1}{2} + \frac{\square}{20}$

$\frac{1}{2} + \frac{\square}{9}$

$\frac{1}{2} + \frac{\square}{12}$

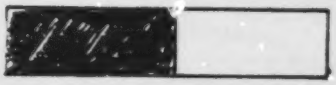
6-12-TR4

Name _____

Circle the fraction that tells about how much is shaded.

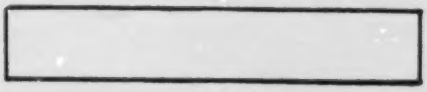
1. $\frac{2}{13}$ $\frac{5}{9}$ $\frac{13}{15}$

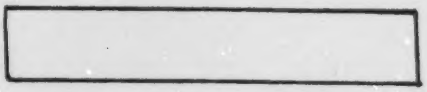
2. $\frac{2}{7}$ $\frac{9}{16}$ $\frac{7}{8}$

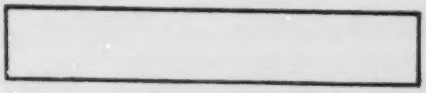
3.  $\frac{3}{19}$ $\frac{6}{11}$ $\frac{15}{17}$

4. $\frac{3}{18}$ $\frac{3}{7}$ $\frac{3}{5}$

Approximate the amounts by shading in the bars.

5. $\frac{1}{8}$ 

6. $\frac{5}{11}$ 

7. $\frac{7}{9}$ 

8. $\frac{6}{15}$

Circle the fractions close to 0, $\frac{1}{2}$, or 1.

9. Close to 0

$\frac{3}{4}$	$\frac{1}{10}$	$\frac{1}{9}$
$\frac{3}{20}$	$\frac{5}{8}$	$\frac{10}{21}$

10. Close to $\frac{1}{2}$

$\frac{7}{12}$	$\frac{1}{8}$	$\frac{5}{6}$
$\frac{6}{13}$	$\frac{7}{15}$	$\frac{12}{15}$

11. Close to 1

$\frac{17}{15}$	$\frac{1}{12}$	$\frac{9}{11}$
$\frac{7}{13}$	$\frac{14}{17}$	$\frac{3}{7}$

12. Make fractions close to 0.

$\frac{\quad}{9}$ $\frac{3}{\quad}$

$\frac{\quad}{5}$ $\frac{2}{\quad}$

13. Make fractions close to $\frac{1}{2}$

$\frac{5}{\quad}$ $\frac{\quad}{9}$

$\frac{\quad}{13}$ $\frac{\quad}{15}$

14. Make fractions close to 1.

$\frac{\quad}{7}$ $\frac{4}{\quad}$

$\frac{3}{\quad}$ $\frac{\quad}{12}$

6-12-p.1

Name: _____

Choose one fraction to make the sum less than or greater than 1.

15. $\frac{3}{11}$ $\frac{5}{11}$ $\frac{7}{11}$ $\frac{9}{11}$ $\quad \frac{1}{2} + \square < 1 \quad \frac{1}{2} + \square > 1$

16. $\frac{3}{8}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\quad \frac{1}{2} + \square < 1 \quad \frac{1}{2} + \square > 1$

17. $\frac{3}{16}$ $\frac{5}{16}$ $\frac{9}{16}$ $\frac{11}{16}$ $\quad \frac{7}{16} + \square < 1 \quad \frac{7}{16} + \square > 1$

18. $\frac{3}{20}$ $\frac{7}{20}$ $\frac{13}{20}$ $\frac{17}{20}$ $\quad \frac{8}{20} + \square < 1 \quad \frac{8}{20} + \square > 1$

19. Circle the examples where the sums are greater than 1.

$\frac{1}{2} + \frac{2}{5}$ $\quad \frac{1}{2} + \frac{3}{5}$ $\quad \frac{1}{2} + \frac{2}{7}$ $\quad \frac{1}{2} + \frac{4}{9}$

$\frac{7}{16} + \frac{1}{10}$ $\quad \frac{9}{16} + \frac{5}{8}$ $\quad \frac{9}{20} + \frac{1}{2}$ $\quad \frac{9}{20} + \frac{2}{15}$

Circle the best estimate.

20.

5 baskets made
11 baskets attempted

made more than $\frac{1}{2}$

made less than $\frac{1}{2}$

21.

8 hits
15 times at bat

hit more than $\frac{1}{2}$

hit less than $\frac{1}{2}$

22.

5 field goals made
9 field goals tried

made more than $\frac{1}{2}$

made less than $\frac{1}{2}$

23.

7 strike-outs
17 batters

struck-out more than $\frac{1}{2}$

struck-out less than $\frac{1}{2}$

6-12-p.2

NSF ESTIMATION
GRADE 6 - LESSON 13

OBJECTIVES: To estimate the sum of fractions.

To estimate the sum of mixed numbers using the front-end strategy.

To adjust estimates.

TEACHER BACKGROUND:

1. Estimating the sum of fractions.

   The ability to identify fractions close to 0, $1/2$ and 1 is used to estimate sums, as shown below.

   $12/13 + 4/9$ ESTIMATE: $1\ 1/2$

   $12/13$ is close to 1 and $4/9$ is close to $1/2$,

   so the sum is about $1\ 1/2$.

   This method is an easy and enjoyable one for students. With the background from the previous lesson, students should pick up quickly on this approach.

2. Estimating the sum of mixed numbers.

   Several approaches can be used to estimate the sum of mixed numbers. The front-end method shown below builds on students' knowledge of front-end addition with whole numbers and the ability to estimate the sum of fractions. It is possible to round each mixed number to a whole number first and then add the rounded numbers. However, the front-end approach is a natural extension of prior work with estimation.

   $2\ 3/8 + 4\ 5/9$

   1) Add the wholes: $2 + 4 = 6$

   2) Estimate the rest:

      $3/8 + 5/9$ is about 1, since each fraction is close to $1/2$.

   3) ESTIMATE: $6 + 1 = 7$

3. Adjusting estimates.

   In many cases it is possible to adjust estimates. For $12/13 + 4/9$ the sum is a little less than $1\ 1/2$, since $12/13$ is less than 1 and $4/9$ is less than $1/2$. So, $1\ 1/2$ is an overestimate and is a little over the

exact sum. So the estimate is written as $1\frac{1}{2}^-$. The lesson encourages students to adjust estimates and it is important to accept a variety of responses, as shown below.

$$6\frac{2}{3} + 5\frac{3}{4}$$

a. about 13

b. a little less than 13

c. about $12\frac{1}{2}$

d. 13^-

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 encourages students to be flexible in choosing numbers with which to work. The 2-step problems and real world setting require students to select numbers that can be handled easily and still produce a realistic estimate. Estimates will vary from "in the ballpark" to quite close to the actual sum.

ANSWERS:

1. $2.75 - $3.50
2. $2.00 (under $2.00 with tax)
3. Yes - 6 cactus plants cost less than $9.00
4. $1.00 - $1.40

ESTIMATING SUMS

TR #2: Use the work at the top to review finding fractions close to 0, $\frac{1}{2}$ and 1. Students are to tell which fractions go in each basket.

Next, develop estimating sums of fractions using the two examples in the middle of the page. Tell students that they should change each fraction to 1, 0 or $\frac{1}{2}$. The answers to the two problems are 2 and 2.

Do the first row of TRY THESE exercises with students. Then have them try the second row independently before discussing their work.

ANSWERS:

$1\frac{1}{2}$	1	$2\frac{1}{2}$
$1\frac{1}{2}$	1	1

TR #3: Develop the three steps of the front-end strategy for mixed numbers. Point out that step 2 uses the work that was on Transparency 2. Do the first row of TRY THESE exercises with students. Then have them do the remaining ones independently before discussing the exercises.

ANSWERS:

18	$11\frac{1}{2}$
$16\frac{1}{2}$	19
9	$7\frac{1}{2}$

ADJUSTING THE ESTIMATES

TR #4: Begin by pointing out that it is natural to indicate how the estimate relates to the answer. Remind them that one way of doing this is to write a + or - after the estimate. Then develop the three examples presented on the transparency. Discuss the TRY THESE exercises with the students.

ANSWERS: 3^- 8

4^+ 8^-

USING THE EXERCISES:

ANSWERS:

1. about 2	2. about 1	3. about 2
4. about 1	5. about $1\ 1/2$	
6. $1\ 1/2$	7. 3	8. 3
9. 1	10. 2	11. 2
12. $1\ 1/2$	13. 2	14. 1
15. over 6	16. under 10	
17. under 8	18. over 7	
19. 12^- $(11\ 1/2)$	20. $8\ 1/2$ (8 - 9)	21. 2^-
22. 10^+ $(10\ 1/2)$	23. 16^-	24. 4
25. 8	26. 9^+ $(9\ 1/2)$	
27. 8	28. 9^+ $(9 - 9\ 1/2)$	
29. 11^+ $(11 - 11\ 1/2)$		30. 15 (15^-)

GET YOUR MIND IN GEAR

TWO-STEP PROBLEMS

When numbers seem "grubby," look for nice numbers to work with.

	5" PLANT
COLEUS	53¢
SWEDISH IVY	75¢
PHILODENDRON	99¢
CACTUS	$1.43

KAREN BOUGHT 3 SWEDISH IVY AND 1 COLEUS. ABOUT HOW MUCH DID SHE SPEND? _____

DAN BOUGHT 3 PHILODENDRON PLANTS WITH A $5 BILL. ABOUT HOW MUCH CHANGE SHOULD HE GET? _____

PHYLLIS HAS $10. IF SHE BUYS 6 CACTUS PLANTS, CAN SHE ALSO BUY 1 SWEDISH IVY? _____

PETE, TONY AND CARLOS BOUGHT ONE OF EACH PLANT. THEY SPLIT THE COST EQUALLY. ABOUT HOW MUCH DID EACH PAY? _____

6-13-TR1

ESTIMATING SUMS

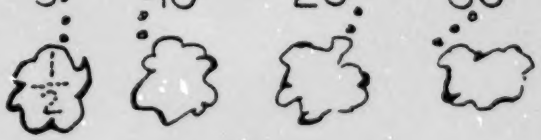
$\frac{4}{5}$	$\frac{1}{14}$	$\frac{7}{8}$	$\frac{6}{14}$
$\frac{1}{15}$	$\frac{2}{47}$	$\frac{5}{9}$	$\frac{6}{7}$
$\frac{3}{100}$	$\frac{5}{12}$	$\frac{10}{13}$	$\frac{7}{16}$

Estimate Sums

$\frac{12}{13} + \frac{7}{8} + \frac{1}{5}$

About ____

$\frac{3}{5} + \frac{9}{10} + \frac{1}{20} + \frac{16}{30}$



About ____

TRY THESE

$\frac{4}{9} + \frac{11}{12}$

$\frac{7}{15} + \frac{6}{11}$

$\frac{3}{4} + \frac{14}{15} + \frac{5}{9}$

$\frac{12}{21} + \frac{7}{8}$

$\frac{19}{41} + \frac{6}{13}$

$\frac{2}{19} + \frac{1}{9} + \frac{14}{15}$

6-13-TR2

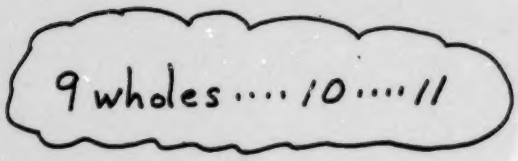
USE FRONT-END ESTIMATION ON FRACTIONS

$$4\frac{7}{8} + 2\frac{1}{9} + 3\frac{11}{12}$$

Add the wholes: $4 + 2 + 3 = 9$

Estimate the rest:

Put it together:



Estimate: 11

TRY THESE

$6\frac{3}{4} + 9\frac{2}{3} + 1\frac{1}{5}$ $\qquad$ $2\frac{3}{4} + 1\frac{9}{10} + 6\frac{4}{7}$

$2\frac{3}{95} + 6\frac{9}{10} + 7\frac{6}{13}$ $\qquad$ $3\frac{8}{9} + 7\frac{13}{15} + 6\frac{3}{4}$

$1\frac{1}{3} + 6\frac{1}{4} + 1\frac{5}{6}$ $\qquad$ $2\frac{1}{7}$ $\quad$ $3\frac{4}{9}$ $\quad$ $1\frac{10}{11}$

6-13-TR3

ADJUSTING

Sometimes it's easy to adjust . . .

$\frac{12}{13} + \frac{7}{8}$

almost 1 | almost 1

Estimate: almost 2

OR 2⁻

$3\frac{4}{5} + 1\frac{7}{8} + \frac{9}{10}$

1⁻ | 1⁻ | 1⁻

4 wholes, ... 5 ... 6 ... 7

Estimate: under 7

OR 7⁻

$1\frac{1}{9} + 2\frac{3}{4} + 3\frac{1}{10}$

0 | 1 | 0

Estimate: 7

Some rounded up ... and some rounded down. I can't tell how to adjust.

TRY THESE

$\frac{7}{9} + \frac{14}{15} + \frac{9}{10}$

$2\frac{1}{4} + 3\frac{9}{10} + 1\frac{4}{5}$

$2\frac{1}{13} + 1\frac{1}{3} + 1\frac{1}{8}$

$1\frac{3}{4} + 2\frac{19}{20} + 2\frac{6}{7}$

6-13-TR4

Name_____

Circle the best estimate.

1. $\frac{13}{15} + \frac{5}{6}$ about 1 about 1½ about 2

2. $\frac{23}{25} + \frac{1}{9}$ about 1 about 1½ about 2

3. $\frac{5}{7} + \frac{9}{16} + \frac{7}{15}$ about 1 about 2 about 3

4. $\frac{1}{10} + \frac{1}{12} + \frac{7}{9}$ about 1 about 1½ about 2

5. $\frac{5}{8} + \frac{6}{11} + \frac{11}{20}$ about 1½ about 2½ about 3

Estimate.

6. $\frac{4}{7} + \frac{7}{8}$ Estimate:_____

7. $\frac{14}{15} + \frac{9}{11} + \frac{9}{10}$ Estimate:_____

8. $\frac{13}{16} + \frac{1}{8} + \frac{13}{14} + \frac{13}{15}$ Estimate:_____

9. $\frac{3}{8} + \frac{9}{16}$ Estimate:_____

10. $\frac{7}{16} + \frac{5}{11} + \frac{9}{10}$ Estimate:_____

11. $\frac{4}{9} + \frac{1}{12} + \frac{17}{18} + \frac{7}{13}$ Estimate:_____

12. $\frac{3}{5} + \frac{9}{10}$ Estimate:_____

13. $\frac{8}{9} + \frac{5}{6}$ Estimate: _____

14. $\frac{1}{10} + \frac{3}{17} + \frac{17}{20}$ Estimate:_____

Circle the best estimate.

15. $3\frac{7}{8} + 2\frac{3}{10}$ over 6 / under 6

16. $6\frac{1}{5} + 3\frac{1}{2}$ over 10 / under 10

17. $5\frac{3}{8} + 2\frac{1}{4}$ over 8 / under 8

18. $6\frac{5}{8} + \frac{5}{8}$ over 7 / under 7

Name _____

Estimate. Use the front-end method. Write a + or - to adjust your estimate when you can.

19. $7\frac{7}{8} + 3\frac{11}{13}$

Estimate: _____

20. $5\frac{9}{16} + 2\frac{13}{24}$

Estimate: _____

21. $\frac{12}{13} + \frac{8}{9}$

Estimate: _____

22. $6\frac{2}{9} + 4\frac{1}{10}$

Estimate: _____

23. $8\frac{5}{11} + 7\frac{6}{13}$

Estimate: _____

24. $2\frac{8}{9} + 1\frac{1}{5}$

Estimate: _____

25. $3\frac{1}{8} + 2\frac{7}{9} + 1\frac{13}{14}$

Estimate: _____

26. $6\frac{1}{12} + 2\frac{1}{9} + 1\frac{2}{15}$

Estimate: _____

Estimate the total for each.

27.

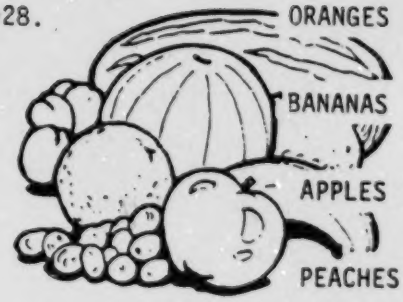
BEEF $3\frac{1}{16}$ pounds

PORK $2\frac{7}{8}$ pounds

CHICKEN $2\frac{1}{10}$ pounds

About _____ pounds of meat

28.



ORANGES $2\frac{1}{8}$ pounds

BANANAS $2\frac{2}{15}$ pounds

APPLES $2\frac{1}{10}$ pounds

PEACHES $3\frac{1}{9}$ pounds

About _____ pounds of fruit

29.

CASHEWS $2\frac{5}{8}$ pounds

$3\frac{9}{16}$ pounds

$2\frac{7}{8}$ pounds

$2\frac{1}{8}$ pounds

About _____ pounds of nuts

30.

SWISS CHEESE $5\frac{1}{16}$ ounces

CHEDDAR CHEESE $4\frac{7}{8}$ ounces

MUNSTER CHEESE $4\frac{9}{10}$ ounces

About _____ ounces of cheese

NSF ESTIMATION
GRADE 6 - LESSON 14

OBJECTIVES: To estimate a fractional part of a number.

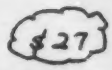
To adjust estimates.

TEACHER BACKGROUND:

1. First students review finding a fractional part of a whole. The idea of finding $1/3$ of 18 is related to splitting 18 into 3 equal parts, or dividing 18 by 3.

$1/3$ of 18

2. To estimate a fractional part, the idea of compatible numbers is used. Since 26 is close to 27, and 27 and 3 are compatible numbers, the estimate is found by dividing 27 by 3.



$1/3$ of $26.25

ESTIMATE: $9.00

3. Again it is natural to adjust the estimate by writing + or - after the estimate. Since $26.25 was rounded up, the answer to $1/3$ of $26.25 is less than $9.00. So $9.00 is an overestimate.

ADJUSTED ESTIMATE: $9.00-

The estimation skill taught in this lesson is one that has wide social usefulness and builds a foundation for later work with estimating a percent of a number.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

Students get practice in adjusting a "ballpark" estimate to get one that is closer on Transparency #1. For each problem have students look at the ballpark estimate and then try to adjust it to get a closer estimate. Have students describe their thinking processes.

ANSWERS: 1. $12.50 - $13.00 2. 600 - 800

3. $20,000 - $26,000 4. $2.50 - $2.90

5. $5.50 - $6.50

ESTIMATE A FRACTIONAL PART

TR #2: The meaning of finding a fractional part is reviewed. Present the problem at the top, then link the idea of finding $1/3$ of 18 to dividing 18 by 3. The link is the idea of splitting 18 into 3 equal parts. Do the first two of TRY THESE exercises with students. Each time have them restate the problem using division.

ANSWERS: 10; 25; 20; 12; 33; 100

8; 30; 200

TR #3: Present the problem at the top and the solution shown. Review with students the idea of compatible numbers used in division. Then discuss the TRY THESE exercises with students. Remind them to choose nice numbers with which to work.

ANSWERS:	$25.00	$20.00	$15.00
		$ 7.00	$20.00
	$22.00	$200.00	$ 5.00

ADJUSTING ESTIMATES:

TR #4: Present the development of the idea of adjusting. Point out that since $117.95 was rounded up, the estimate is too high (an over-estimate). Therefore, the estimate is adjusted downward by writing a - after the estimate. Do the first row of the TRY THESE exercises with students. Then let them try the remaining ones independently before discussing them.

ANSWERS:	$10.00^{+}	$100^{-}	$25.00^{-}	($24.00^{-})
	$ 2.00^{+}	$22^{+}	$60^{+}	
	$ 3.00^{+}	$3.00^{+}	$50^{-}	

USING THE EXERCISES:

The key exercises are 1-25. At the bottom of page 2 is a set of THINK IT THROUGH exercises dealing with mental computation for multiplying a mixed number and a whole number.

ANSWERS:

1.	30	2.	6	3.	7	4.	40	5.	200	6.	100
7.	25	8.	100	9.	60						
10.	20; 5			11.	36; 12			12.	32; 8		
13.	$200; $100			14.	$88; $22			15.	$30; $6		
16.	20 - 22			17.	$38 - $40						
18.	100 - 110			19.	$15 - $16						
20.	9 - 10			21.	$30 - $31						
22.	$10 - $11			23.	$45 - $50						
24.	$4 - $5			25.	$28 - $30						
26.	over $30			27.	over $33	28.	over $25			29.	under $50
30.	over $50			31.	over $30						
32.	4 - 5^{-}			33.	16 - 18	34.	9 - 10			35.	18 - 22

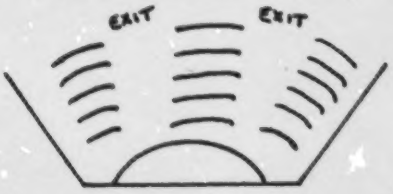
THINK IT THROUGH:

1. 21
2. 18
3. 10
4. 14
5. 26
6. 33

GET YOUR MIND IN GEAR

Are you a good adjuster? See if you can Get Closer with these.

	In The Ballpark	Getting Closer
HOW MUCH FOR 2 ADULTS AND 2 CHILDREN?	$12.00	

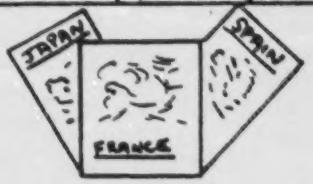


THEATER

25 ROWS OF SEATS, 29 SEATS IN A ROW. HOW MANY SEATS IN ALL?	900	

FUND DRIVE	
Goal:	$160,000
To Date:	$134,750

HOW MUCH MORE TO GO?	$30,000	



POSTERS

3 FOR $8.50. HOW MUCH FOR EACH ONE?	$ 3.00	

LUNCH SPECIAL

Soup	$1.19
Sandwich	$3.49
Pie	$ .99
Milk	$ .49

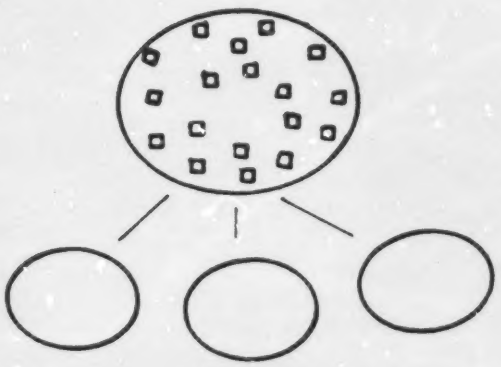
HOW MUCH FOR LUNCH?	$ 5.00	

6-14-TR1

FIND A PART

THE ART CLUB HAD 18 MEMBERS ENTER AN ART CONTEST. $\frac{1}{3}$ OF THEM WON PRIZES. HOW MANY WON PRIZES?

FIND $\frac{1}{3}$ OF 18



$\frac{1}{3}$ OF 18 = ____

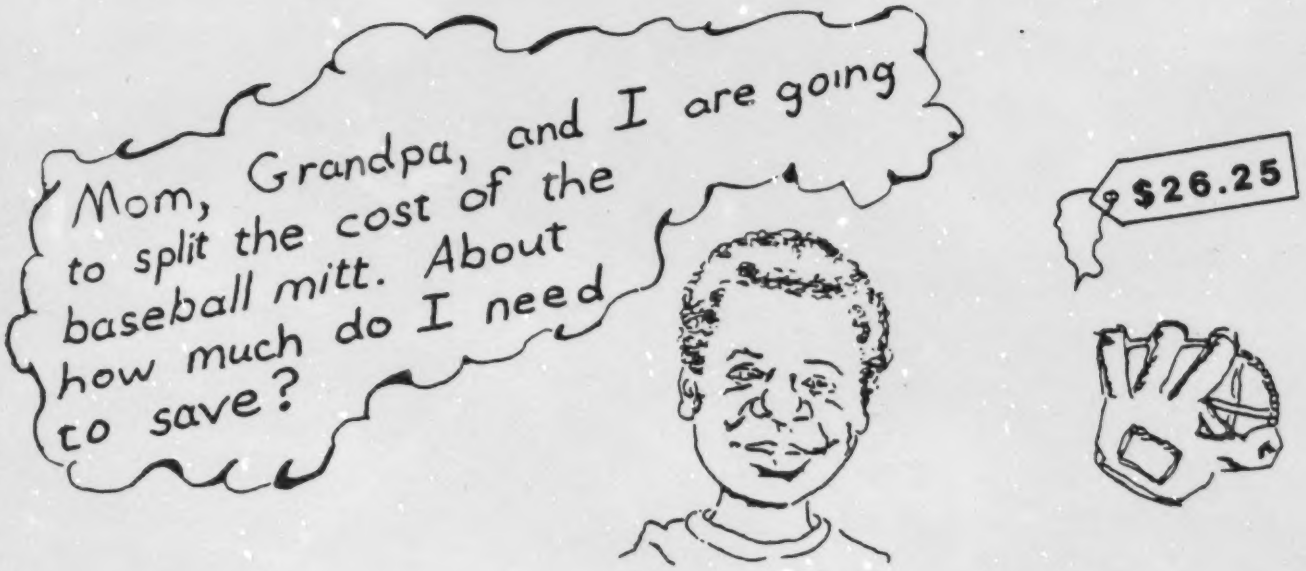
To find ⅓ of 18. I think about 3 equal parts. That means I could Think 18 ÷ 3 to find the number in each part!

Try these!

$\frac{1}{3}$ OF 30	$\frac{1}{2}$ OF 50	$\frac{1}{4}$ OF 80
$\frac{1}{2}$ OF 24	$\frac{1}{3}$ OF 99	$\frac{1}{2}$ OF 200
$\frac{1}{4}$ OF 32	$\frac{1}{4}$ OF 120	$\frac{1}{3}$ OF 600

6-14-TR2

USE COMPATIBLE NUMBERS TO FIND A PART



Estimate: $\frac{1}{3}$ OF $26.25

Use a compatible number:

Then divide: To find $\frac{1}{3}$ of 27, divide by 3

About _____

TRY THESE!

$\frac{1}{2}$ OF $49.19	$\frac{1}{3}$ OF $61	$\frac{1}{2}$ OF $29.95
	$\frac{1}{4}$ OF $27.50	$\frac{1}{4}$ OF $81.50
$\frac{1}{4}$ OF $87	$\frac{1}{2}$ OF $398	$\frac{1}{3}$ OF $14.20

6-14-TR3

Lamp **ADJUSTING**

$117.95

SALE

$\frac{1}{3}$ OFF

To estimate:

$120

$\frac{1}{3}$ OF $117.95

ABOUT $40

To adjust: THINK...

Let's see .... I rounded $117.95 up, so $40 is an overestimate. I'll need to adjust down... $40-

TRY THESE

$\frac{1}{3}$ OF $31.95	$\frac{1}{4}$ OF $395	$\frac{1}{2}$ OF $47.98
$\frac{1}{2}$ OF $ 4.35	$\frac{1}{3}$ OF $ 67	$\frac{1}{4}$ OF $242
$\frac{1}{4}$ OF $12.59	$\frac{1}{2}$ OF $6.18	$\frac{1}{3}$ OF $148

6-14-TR4

Name_____

Write the answer.

1. $\frac{1}{2}$ of 60 = _____
2. $\frac{1}{4}$ of 24 = _____
3. $\frac{1}{3}$ of 21 = _____
4. $\frac{1}{4}$ of 160 = _____
5. $\frac{1}{2}$ of 400 = _____
6. $\frac{1}{3}$ of 300 = _____
7. $\frac{1}{4}$ of 100 = _____
8. $\frac{1}{3}$ of 300 = _____
9. $\frac{1}{8}$ of 480 = _____

Rewrite each problem using a compatible number. Then use the compatible number to estimate the answer.

10. $\frac{1}{4}$ of 21 — $\frac{1}{4}$ of _____ — Estimate: _____
11. $\frac{1}{3}$ of 35 — $\frac{1}{3}$ of _____ — Estimate: _____
12. $\frac{1}{4}$ of 31 — $\frac{1}{4}$ of _____ — Estimate: _____
13. $\frac{1}{2}$ of \$197 — $\frac{1}{2}$ of _____ — Estimate: _____
14. $\frac{1}{4}$ of \$89.25 — $\frac{1}{4}$ of _____ — Estimate: _____
15. $\frac{1}{5}$ of \$31.59 — $\frac{1}{5}$ of _____ — Estimate: _____

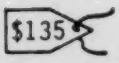
Estimate. Adjust your estimate by putting a + or - by your answer.

16. $\frac{1}{3}$ of 65 Estimate: _____
17. $\frac{1}{4}$ of \$154 Estimate: _____
18. $\frac{1}{2}$ of 215 Estimate: _____
19. $\frac{1}{2}$ of \$31.50 Estimate: _____
20. $\frac{1}{4}$ of 37 Estimate: _____
21. $\frac{1}{3}$ of \$92 Estimate: _____
22. $\frac{1}{3}$ of \$31.59 Estimate: _____
23. $\frac{1}{4}$ of \$192 Estimate: _____
24. $\frac{1}{7}$ of \$33.39 Estimate: _____
25. $\frac{1}{5}$ of \$149.95 Estimate: _____

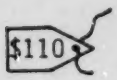
Name _____

Circle the best estimate. How much is saved?

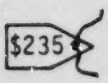
$\frac{1}{4}$ OFF!

26. 

under $30
over $30

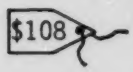
28. 

under $25
over $25

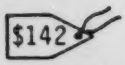
30. 

under $50
over $50

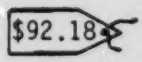
$\frac{1}{3}$ OFF!

27. 

under $33
over $33

29. 

under $50
over $50

31. 

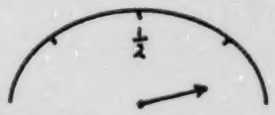
under $30
over $30

Estimate the gallons of gas in each tank. Adjust your estimate by writing a + or - next to your answer.

32. A full tank holds 10 gal.

Estimate: _____

33. A full tank holds 19 gal.

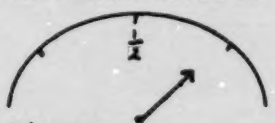


Estimate: _____

34. A full tank holds 38 gal.

Estimate: _____

35. A full tank holds 29 gal.



Estimate: _____

Find the exact answer.

$3\frac{1}{2} \times 10 =$ ____

Think: 3 x 10 is 30. $\frac{1}{2}$ of 10 is 5. So the answer is 30+5 or 35.

1. $2\frac{1}{3} \times 9 =$ _____

2. $2\frac{1}{4} \times 8 =$ _____

3. $4 \times 2\frac{1}{2} =$ _____

4. $6 \times 2\frac{1}{3} =$ _____

5. $3\frac{1}{4} \times 8 =$ _____

6. $5\frac{1}{2} \times 6 =$ _____

NSF ESTIMATION
GRADE 6 - LESSON 15

OBJECTIVE: To estimate answers for addition and subtraction of decimals.

TEACHER BACKGROUND:

Estimation with decimals is developed for two cases: 1) estimation with "smaller" numbers (numbers between 0 and 10 or in the low tens) and 2) estimation with "larger" numbers (numbers with 2 or more digits). The strategies employed were taught with addition and subtraction of whole numbers and are extended to decimals. Students are encouraged to select a strategy that works well for them.

Decimal estimation is useful in reducing errors when working with different numbers of decimal places. Students often line up the right-hand digits, ignoring the decimal places.

$3.8 + .27 + 1.5$

$$\begin{array}{r} 3.8 \\ .27 \\ +\ 1.5 \\ \hline \end{array}$$

1. Estimation with Smaller Numbers

   A. FRONT-END

$0.2 + 2.8 + 9.7 + 7$	$15.3 - 7.8$
1) $0 + 2 + 9 + 7 = 18$	1) $15 - 7 = 8$
2) $.2 + .8 + .7$ is between 1 and 2	2) .3 is less than .8
3) ESTIMATE: 19^{+}, 19.5, etc.	3) ESTIMATE: 8^{-}, 7.5, etc.

   B. ROUNDING

$0.2 + 2.8 + 9.7 + 7$	$15.3 - 7.8$
1) $0 + 3 + 10 + 7 = 20$	1) $15 - 8 = 7$
2) ESTIMATE: 20	ESTIMATE: 7

2. Estimation with Larger Numbers

Students are encouraged to choose numbers that are easy to use.

$92.7 + 48.5 + 63.72$	$14.27 + 6.83 + 34.6 + 7.2$
1) $100 + 50 + 60$	1) $20 + 40$
2) ESTIMATE: 210	2) ESTIMATE: 60

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 presents a real-world setting involving multi-step problems. After students look at the chart, have them estimate for each problem. Let several students give their estimates and describe how they were determined. Accept a variety of approachs and estimates.

ANSWERS:
1. $17 - $18
2. Yes ($4.00 - $4.50)
3. $2 - $3
4. 5 boxes (4 boxes with tax)

ESTIMATE SUMS AND DIFFERENCES

TR #2: Present the information and the problem at the top. Let students suggest how they might estimate. Then present the FRONT-END and ROUNDING strategies. The front-end estimate can be refined to 19.5 or 19.7.

Do the first row of TRY THESE exercises with students. Then let them try the remaining ones independently.

ANSWERS:

10 - 11	8 - 9	4 (4^+)
14 - 15.5	8 - 9	9 - 10
17 - 18	18 - 19	21^+ - 22

TR #3: Here students use estimation skills to check for sensible answers. The emphasis is placed on deciding whether the answer is "in the ballpark". They do not need to find if the computation itself is correct. Discuss the example at the top. Then have them quickly look over the test to spot the answers that are not sensible.

ANSWERS: Not sensible #3, #4, #6, #7, #8, #10

TR #4: The focus is on looking for numbers that "go together" or that are "easy to work with". Discuss the three approaches shown for the example at the top. Then discuss the TRY THESE exercises with students.

ANSWERS:

70 - 75	$60 - 70^-$
170 - 180	20^+ - 30
180 - 200	$190 - 200^-$
180 - 200	75^+ - 90
60 - 65	26^+

USING THE EXERCISES:

The exercises are a straight-forward application of the content of the lesson. Make sure that students understand the chart on page 1.

ANSWERS:

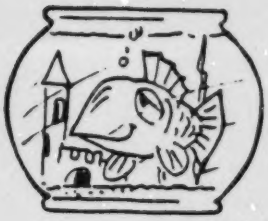
1. 45	2. 30	3. 50
4. 10	5. 350	
6. 117 - 120	7. 110 - 115	8. 130 - 135
9. 140 - 145	10. 140 - 150	
11. under 45	12. under 100	13. over 40
14. over 20	15. under 250	16. over 400

Exercises 17 - 26: The answers that are not sensible are:

#18, #19, #21, #23, #26

27. more	28. 4 - 5 seconds	29. 5 - 6 seconds
30. 60 - 65	31. 22 - 24	32. 60 - 65
33. 340 - 350	34. 22 - 24	

CANARY $14.99 EA.
GUPPIES 2 FOR 49¢
GOLD FISH $1.29 EA.
BIRD SEED $ .89 FOR 12 OZ.
FISH FOOD $ .42 FOR 2 OZ.



CARLIN BOUGHT A CANARY, 4 GUPPIES, AND 2 BOXES OF BIRD SEED, ABOUT HOW MUCH DID SHE SPEND ?

JUAN WAS GIVEN $5 FOR HIS BIRTHDAY. CAN HE BUY 3 GOLDFISH AND A BOX OF FISH FOOD?

IRENE BOUGHT 8 BOXES OF BIRD SEED. SHE GAVE THE CLERK A $10 BILL. ABOUT HOW MUCH CHANGE SHOULD SHE GET?

ERIC HAS $20. HE WANTS TO BUY A CANARY AND SOME BIRD SEED. HOW MANY BOXES OF BIRD SEED CAN HE BUY?

6-15-TR1

ESTIMATING WITH DECIMALS

BEAN PLANT GROWTH

WEEK	NEW GROWTH
1	0.0 cm
2	0.2 cm
3	2.8 cm
4	9.7 cm
5	7.0 cm

WHAT WAS THE TOTAL GROWTH OVER 5 WEEKS?

0.2 + 2.8 + 9.7 + 7

Use Front-end!

Add the wholes:
0.2 + 2.8 + 9.7 + 7
.... 18
Estimate the rest:
.2 + .8 + .7 → over 1
Put it together → 19+ cm

Use Rounding to Wholes!

0.2 + 2.8 + 9.7 + 7
↓ 0 ↓ 3 ↓ 10 ↓ 7
about 20 cm

TRY THESE

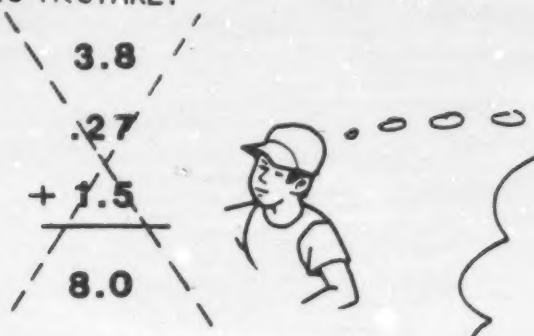
8 + 2.73 + 0.12	3.3 + 4.9 + 0.7	10.2 − 6
3.4 + 5 + 6.71	2.1 + 4.5 + 2	16.8 − 7.43
12.82 + 5	10.23 + 8.7	24.6 − 3.2

6-15-TR2

IS THE ANSWER SENSIBLE?

3.8 + .27 + 1.5

DO YOU EVER MAKE THIS MISTAKE?



USE ESTIMATION TO LOOK BACK AT YOUR ANSWERS.

3 and 1 make 4 wholes. The rest will make the answer go a little over 5.

8.0 is not sensible!

TRY THESE:

TEST: FIND THE EXACT ANSWERS. NAME: Jack

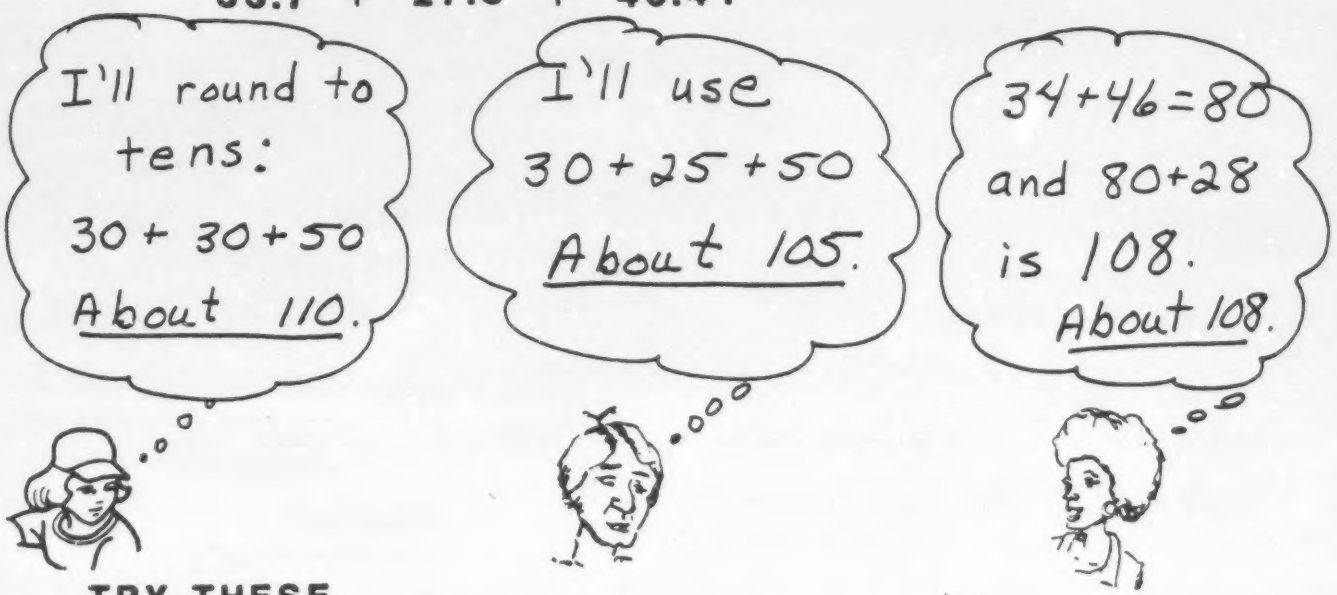
(1) 3.8 + .27 + 1.5	5.57	(2) 4.6 + 8.22	12.82
(3) 6.2 + .34 + .72	16.8	(4) 9.001 + 1.1	11.101
(5) .8 + 6.7 + 2.3	9.8	(6) 1 + 8.4	8.5
(7) 9.7 − 3	6.4	(8) 6.7 − 12	5.5
(9) 5.67 − 1.2	4.47	(10) 3 − 2.67	2.64

6-15-TR3

ESTIMATION WITH LARGER NUMBERS

When estimating with larger numbers, use whole numbers that are easy for you to use.

33.7 + 27.8 + 46.41



TRY THESE

16.98 + 24.3 + 32.7

38.65 + 41.52 + 99.01

39.01 + 101.4 + 59.52

44.82 + 51.7 + 89.66

14.9 + 17.008 + 32.56

84.7 - 16.92

46.9 - 23.008

248.7 - 52.5

157 - 72.03

28.19 - 2.018

6-15-TR4

Name _____

Circle the best estimate.

1.	4.201 + 16.3 + 24.82	7	45	82
2.	33.21 - 1.6	2	20	30
3.	33.8 + 14 + 7.9	24	50	110
4.	1.9 + 6.4 + .8 + 2.1	10	38	100
5.	327 + 24.009	100	150	350

This chart shows how much some third grade students grew during the year. Estimate their height in the spring.

	HEIGHT IN FALL	GREW	HEIGHT IN SPRING
6. Jenny	112.5 cm	7.25 cm	
7. Miquel	98.2 cm	12.5 cm	
8. Janice	121.75 cm	11.5 cm	
9. Elise	134.25 cm	8.3 cm	
10. Karl	128.5 cm	16 cm	

Circle the best estimate.

11. 28.67 + 14.36

over 45 under 45

12. 145.6 - 53.71

over 100 under 100

13. 21.0008 + 16 + 7.2

over 40 under 40

14. 43.08 - 21.65

over 20 under 20

15. 245.8 + .287

over 250 under 250

16. 452.1 - 48

over 400 under 400

6-15-p.1

Name _____

Karen finished her quiz early. Help her look back to see if the answers are sensible. Make an "X" on the answers that are not sensible.

17. 24.6 - 13.82 = 10.78

18. 9.6 + 14 + 8.7 = 19.7

19. 43.9 - .36 = 40.3

20. 37.2 + 19.08 = 56.28

21. 46.8 - 27 = 42.1

22. 8.62 + 21 + 4.3 = 33.92

23. 82.7 - 14.32 = 19.49

24. 73.5 + 24.16 = 97.66

25. 82.7 - 14.32 = 68.38

26. 8 + 4.2 + 16.32 = 16.82

27. In 1896, Thomas Burke of the U.S.A won the Olympic gold medal for the 400 meter race. His time was 54.2 seconds. The Olympic record set by Lee Evans of the USA in 1968 was 43.8 seconds.

Evans was _____ 10 seconds faster.
(more than/less than)

28. Irena Szcwinska of Poland made the women's record for the 400 meter race with 49.29 seconds in 1976.

About how much faster was Szcwinska than Burke?

29. About how much faster was Evans than Szcwinska?

Estimate.

30. 35.87 + 4 + 1.75 + 21 Estimate: _____

31. 24.8 - 1.67 Estimate: _____

32. 43 + .8 + 17.2 Estimate: _____

33. 367 - 24.98 Estimate: _____

34. .099 + 12.8 + 4 + 6.7 Estimate: _____